CS 3500 Section A, Spring 2000

# Homework 5 – due Tuesday, Feb. 22

**To allow for prompt posting of solutions, no late homeworks will be accepted this time.**

**Problem 1** Let $M_0$ be a *fixed* Turing machine and $w_0$ be a *fixed* string. Is it decidable whether $M_0$ halts on $w_0$ or not? Is it decidable whether $M_0$ always halts? Why or why not?

**Problem 2** **(a)** Prove that, if $L_1$ and $L_2$ are Turing decidable languages, then $L_1 - L_2$ is also Turing decidable. **(b)** Give two Turing recognizable languages $L_1$ and $L_2$ such that $L_1 - L_2$ is not Turing recognizable.

**Problem 3** Prove that the language

$$\mathsf{HALT\text{-}ON\text{-}BLANK\text{-}TAPE}_{TM} = \{< M > \mid M \text{ is a TM and } M \text{ halts when started on the blank tape}\}$$

is not decidable.

**Hint:** Show that $\mathsf{HALT}_{TM} = \{< M, w > \mid M \text{ is a TM, } w \text{ is a string over } M\text{'s input alphabet, and } M \text{ halts on } w\}$ reduces to $\mathsf{HALT\text{-}ON\text{-}BLANK\text{-}TAPE}_{TM}$. Don't get confused by the fact that $\mathsf{HALT\text{-}ON\text{-}BLANK\text{-}TAPE}_{TM}$ reduces trivially to $\mathsf{HALT}_{TM}$, we need the other direction. The idea is to show that we can construct for each pair $< M, w >$ a Turing machine that halts on the blank tape if and only if $M$ halts on $w$.

**Problem 4** Prove that the language

$$\mathsf{ALWAYS\text{-}HALT}_{TM} = \{< M > \mid M \text{ is a TM and } M \text{ halts on all inputs}\}$$

is not decidable.
**Hint:** Reduce $\mathsf{HALT\text{-}ON\text{-}BLANK\text{-}TAPE}_{TM}$ to $\mathsf{ALWAYS\text{-}HALT}_{TM}$.

**Problem 5** Prove that the language

$$\mathsf{CONTEXT\text{-}FREE}_{TM} = \{< M > \mid M \text{ is a TM and } L(M) \text{ is context-free}\}$$

is not decidable.

**Problem 6** Prove that the language

$$\mathsf{FINITE}_{TM} = \{< M > \mid M \text{ is a TM and } L(M) \text{ is finite}\}$$

is not decidable.

**Problem 7** Problem 5.12, page 195 of *Sipser*.