Handouts: Lines and Circles

Design of Line Algorithms

Basic Math Review

Slope-Intercept Formula For Lines

Given a third point on the line:

P = (X,Y)

Slope = **(Y - Y1)/(X - X1)**

= **(Y2 - Y1)/(X2 - X1)**

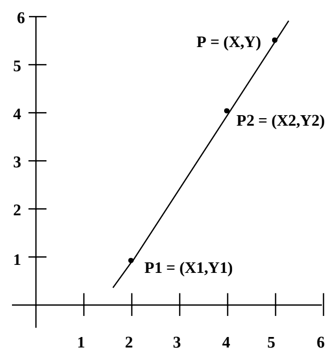
Solve for Y

Y = [(Y2-Y1)/(X2-X1)]X +

[-(Y2-Y1)/(X2-X1)]X1 + Y1

or

Y = mx + b



$$\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{Y2-Y1}{X2-X1}$$

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Other Helpful Formulas

Length of line segment between P1 and P2:

L = ¦ [(X2-X1)2 + (Y2-Y1)2]

Midpoint of a line segment between P1 and P3:

P2 = ((X1+X3)/2 , (Y1+Y3)/2)

Two lines are perpendicular iff:

M1 = -1/M2

Parametric Form of the Equation of a 2D Line

Given points P1 = (X1, Y1) and P2 = (X2, Y2)

X = X1 + t(X2-X1)

Y = Y1 + t(Y2-Y1)

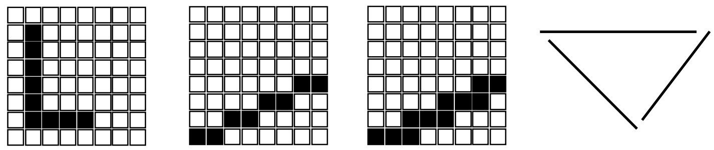
When

t = 0 we get (X1,Y1)

t = 1 we get (X2,Y2)

As 0 < t < 1, we get all points between (X1,Y1) and (X2,Y2)

Basic Line Algorithm



Must:

1. Compute integer coordinates of pixels which lie on or near a line.
2. Be efficient.
3. Create visually satisfactory images.
   - Lines should appear straight
   - Lines should terminate accurately
   - Lines should have constant density
   - Line density should be independent of line length and angle
4. Always be defined.

Simple DDA Line Algorithm

{Based on the parametric equation of a line}

```
Procedure DDA(X1,Y1,X2,Y2 :Integer);
Var         Length, I   :Integer;
     X,Y,Xinc,Yinc      :Real;

Begin
     Length := ABS(X2 - X1);
     If ABS(Y2 - Y1) > Length Then
          Length := ABS(Y2-Y1);
     Xinc := (X2 - X1)/Length;
     Yinc := (Y2 - Y1)/Length;
     X := X1;
     Y := Y1;
     For I := 0 To Length Do
     Begin
          Plot(Round(X), Round(Y));
          X := X + Xinc;
          Y := Y + Yinc
     End {For}
End; {DDA}
```

- Creates good lines, but problems ...

DDA Example

Render the line from (6,9) to (11,12):

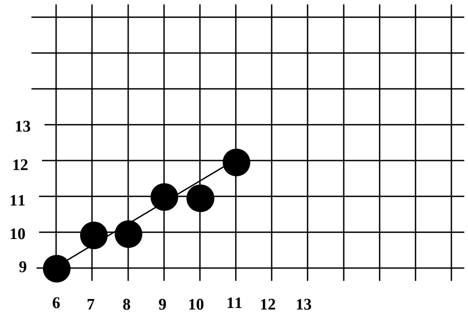
Length := Max of (ABS(11-6), ABS(12-9)) = 5

Xinc := 1

Yinc := 0.6

Values computed are:

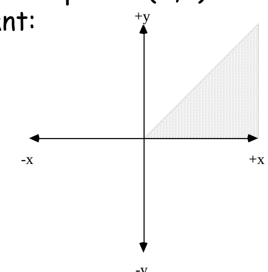
(6,9),
(7,9.6),
(8,10.2),
(9,10.8),
(10,11.4),
(11,12)



Fast Lines Using The Midpoint Method

Assumptions: line between points (0,0) and (a,b) with slope $0 \leq m \leq 1$

i.e. lies in first octant:



Recall: $y = mx + B$ (m is the slope, B is the y-intercept)

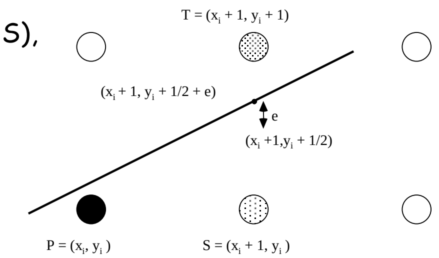
$\Rightarrow m = b/a$ and $B = 0$

$\Rightarrow y = (b/a)x + 0$

$\Rightarrow f(x,y) = bx - ay = 0$

Fast Lines (cont.)

Two choices for next pixel (T or S), want the pixel closer to line!



Assume distance between pixel centers is 1

Midpoint is (xi + 1,yi + 1/2)

e is difference between midpoint and where line crosses between S and T

If e is positive, line crosses above the midpoint and is closer to T

If e is negative, line crosses below the midpoint and is closer to S

⇒ don't need exact value of e

Fast Lines: The Decision Variable

f(xi+1,yi+ 1/2 + e) = b(xi+1) - a(yi+ 1/2 + e) = b(xi + 1) - a(yi + 1/2) -ae
= f(xi + 1, yi + 1/2) - ae = 0

Let di = f(xi + 1, yi + 1/2) = ae; di is known as the decision variable.

Since a ≥ 0, di has the same sign as e.

Algorithm:

If di ≥ 0 then
 Choose T = (xi + 1, yi + 1) as next point
 di+1 = f(xi+1 + 1, yi+1 + 1/2) = f(xi +1+1, yi +1+1/2)
 = b(xi +1+1) - a(yi +1+1/2) = f(xi + 1, yi + 1/2) + b - a
 = di + b - a
else
 Choose S = (xi + 1, yi) as next point
 di+1 = f(xi+1 + 1, yi+1 + 1/2) = f(xi +1+1,yi +1/2)
 = b(xi +1+1) - a(yi +1/2) = f(xi + 1, yi + 1/2) + b
 = di + b

Fast Line Algorithm

Calculate initial value for d_0 directly from f(x,y) at (0,0):

$d_0 = f(0 + 1, 0 + 1/2) = b(1) - a(1/2) = b - a/2$

Algorithm for a line from (0,0) to (a,b) in the first octant is:

```
x := 0;
y := 0;
d := b - a/2;
For i := 0 to a do Begin
        Plot(x,y);
        If d ≥ 0 Then Begin
                x := x + 1;
                y := y + 1;
                d := d + b - a
        End
        Else Begin
                x := x + 1;
                d := d + b
        End
End
```

The only non-integer value is a/2. How can we get rid of it?

Bresenham's Line Algorithm

Generalize for lines beginning at points other than (0,0)

```
Begin {Bresenham for lines with slope between 0 and 1}
    a := ABS(xend - xstart);
    b := ABS(yend - ystart);
    d := 2*b - a;
    Incr1 := 2*(b-a);
    Incr2 := 2*b;
    If xstart > xend Then Begin
            x := xend;
            y := yend
    End
    Else Begin
            x := xstart;
            y := ystart
    End;
    For I := 0 to a Do Begin
            Plot(x,y);
            x := x + 1;
            If d ≥ 0 Then Begin
                    y := y + 1;
                    d := d + incr1
            End
            Else
                    d := d + incr2
    End {For Loop}
End; {Bresenham}
```

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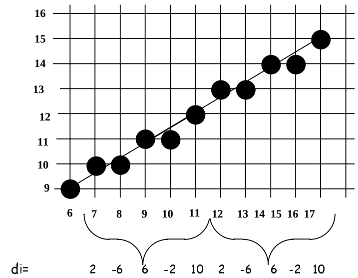
Optimizations

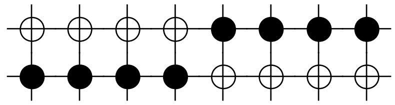
- Detect cycles in the decision variable
  - correspond to a repeated pattern of pixel choices
- Save pattern, reuse if a cycle is detected



Antialiasing

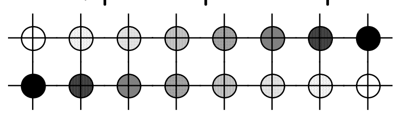
- Aliasing caused by finite addressability of CRT
- Approximation of lines with discrete points can result in a staircase appearance or "Jaggies"
- Desired line

- Aliased rendering of the line



Antialiasing - Solutions

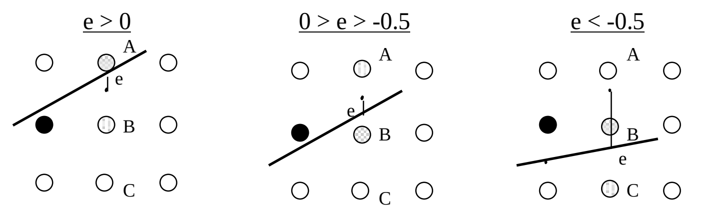
- Aliasing can be smoothed out by using higher addressability.
- Problem: addressability usually fixed
- Solution: intensity is variable, so use it
- ⇒ two adjacent pixels can give impression of point part way between
- ⇒ perceived location of point dependent upon ratio of intensities



- An antialiased line has virtual pixels “located” at the proper addresses

Antialiased Bresenham Lines

- Use the distance (e = di/a) value to determine pixel intensities.
- Three possible cases for the Bresenham algorithm:



e > 0	0 > e > -0.5	e < -0.5
A = 0.5 + e	A = 0.5 + e	A = 0
B = 1 - abs(e+0.5)	B = 1 - abs(e+0.5)	B = 1 - abs(e+0.5)
C = 0	C = 0	C = -0.5 - e

- What about color?