Handouts: Projections

2D Transformations

Rotation About the Origin

sin (A + B) = y2/r

cos (A + B) = x2/r

sin A = y1/r, cos A = x1/r

From the double angle formulas

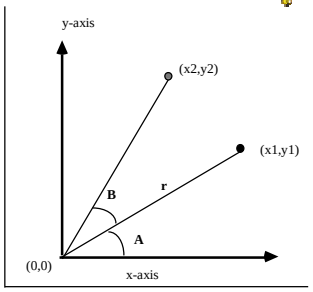
sin (A + B) = sinA cosB + cosA sinB

$\therefore$ y2/r = (y1/r)cosB + (x1/r)sinB

y2 = x1 sinB + y1 cosB

Similarly

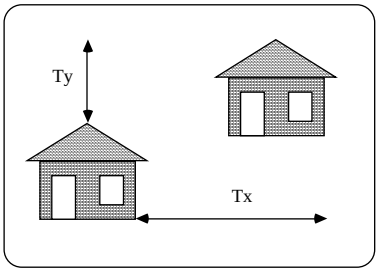
x2 = x1 cosB - y1 sinB



Handouts: Projections

Translations

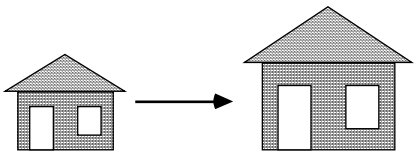
- Translation = moving an object
- Translate object
  - translate each vertex
- Translate point
  - add translation (tx, ty) to vertex (x1, y1)



$x_2 = x_2 + t_x$ $y_2 = y_1 + t_y$

Scaling

- Scaling = changing the size of an object
- Scale object
  - scale each vertex



$x_2 = s_x x_1$ $y_2 = s_y y_1$

- Scale point
  - multiply scale factor (sx, sy) by vertex (x1, y1)

Handouts: Projections

Homogeneous Coordinates

- Represent transformations as matrices
  - easier to manipluate and use
- Use 3 x 3 matrices
  - needed to represent a translation as a matrix operation
- Represent points as 1 x 3 matrices
  - point P = (x, y, 1)

Transforms We've Seen So Far

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

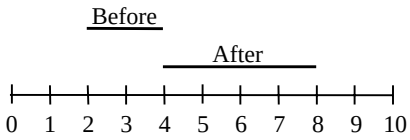
$$S = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & Tx \\ 0 & 1 & Ty \\ 0 & 0 & 1 \end{bmatrix}$$

Handouts: Projections

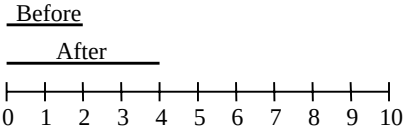
Composite Transformations

- Problem:
  - scale transformation moves the object being scaled
- i.e. scale the line [(2, 1), (4,1)] by 2x



Composite Transforms (cont.)

- Notice: scale line [(0, 1), (2,1)] by 2x
  ⇒ left end does not move



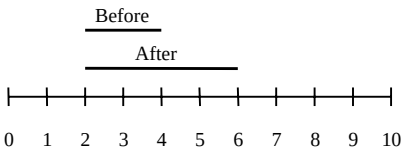
(0,0) is a ***fixed point*** for the scaling transformation

Use composite transformations to create scale transformations with different fixed points

Handouts: Projections

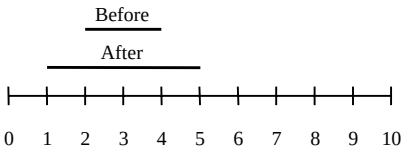
Fixed Point Scaling

- Scale by 2 with fixed point = (2,1)
  - Translate the point (2,1) to the origin
  - Scale by 2
  - Translate origin to point (2,1)



More Fixed Point Scaling

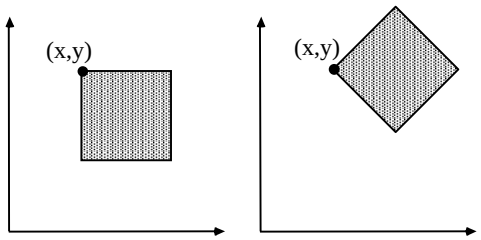
- Scale by 2 with fixed point = (3,1)
  - Translate the point (3,1) to the origin
  - Scale by 2
  - Translate origin to point (3,1)



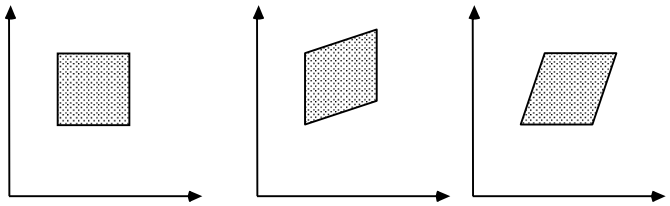
Handouts: Projections

Rotation About a Fixed Point

- Rotation Of ø Degrees About Point (x,y)
  - Translate (x,y) to origin
  - Rotate by ø
  - Translate origin to (x,y)



Shears



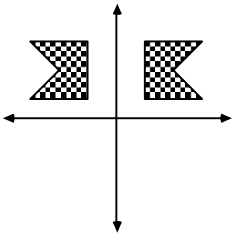
Original Data y Shear x Shear

e.g., GRAPHICS ⇒ x shear ⇒ *GRAPHICS*

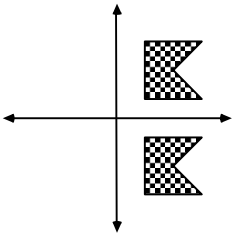
Handouts: Projections

Reflections

Y-axis

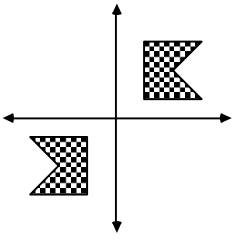


X-axis



More Reflections

Origin



y=x