

Homework 10 – due Thursday, Nov. 16

**Problem 1** Exercise 16.1–1, page 308 of CLR.

**Problem 2** Give examples showing that the following algorithms do *not* always find the optimum matrix-chain multiplication. In other words you must give a number  $n$  and dimensions  $d_i \times d_{i+1}$  for a chain of  $n$  matrices,  $A_1, \dots, A_n$ , such that performing matrix multiplication in the order suggested by the respective algorithm takes more time than performing the multiplication in some other order (which you should also give as part of your solution).

- (a) Multiply matrices from right to left.
- (b) Multiply first two consecutive matrices,  $A_i$  and  $A_{i+1}$ , that take the least amount of time to multiply. Replace the pair  $A_i, A_{i+1}$  by their product, then apply the same algorithm to the resulting chain of  $n - 1$  matrices until a single matrix remains.

**Problem 3** Consider the following (Chomsky normal form) context-free grammar:

$$\begin{aligned} S &\rightarrow RT \\ R &\rightarrow TR \mid a \\ T &\rightarrow TR \mid b \end{aligned}$$

Apply the dynamic programming algorithm presented in class (see also pages 240–241 of Sipser’s book) to decide whether the string  $w = baba$  is generated by the grammar or not.

**Problem 4** Using the dynamic programming algorithm presented in class, construct the optimum binary search tree for a set of five keys  $x_1 < x_2 < x_3 < x_4 < x_5$ , each of which is searched with a probability of 0.1, 0.3, 0.1, 0.1, and 0.4, respectively. (The probability of searching a key  $x \notin \{x_1, \dots, x_5\}$  is zero.)