

Homework 4 – due Tuesday, Sept. 19

**Problem 1** Give context-free grammars that generate the following languages over the alphabet  $\{0, 1\}$ :

- (a)  $\{w \in \{0, 1\}^* \mid w \text{ contains more 1's than 0's}\}$
- (b)  $\{w \in \{0, 1\}^* \mid w \text{ contains twice as many 1's as 0's}\}$
- (c)  $\{0^n 1^m \mid n + m \text{ is even}\}$ .
- (d)  $\{w \in \{0, 1\}^* \mid w \neq w^R\}$

**Problem 2** Which of the following languages over  $\{0, 1\}$  are context-free? Give a context-free grammar for the languages that are context-free, and give a proof for the languages that are not context-free.

- (a)  $\{0^i 1^j \mid j = 2i\}$
- (b)  $\{0^i 1^j \mid j = i^2\}$
- (c)  $\{0^i 1^j \mid i \neq j\}$
- (d)  $\{ww \mid w \in \{0, 1\}^*\}$

**Problem 3** Convert the following CFG into an equivalent CFG in Chomsky normal form (the start variable is  $S$ ):

$$\begin{aligned} S &\rightarrow aaSb \mid T \\ T &\rightarrow bbTa \mid S \mid \varepsilon \end{aligned}$$

**Problem 4** Prove that the language  $\{1^p \mid p \text{ is a prime number}\}$  is not context-free.

**Problem 5** Problem 2.2, page 120 of *Sipser*.

**Problem 6** Problem 2.17 (b), page 121 of *Sipser* (assume part (a) to be true).