

Homework 7 – due Wednesday, Oct. 11

Problem 1 (a) Prove that, if L_1 and L_2 are Turing decidable languages, then $L_1 - L_2$ is also Turing decidable.
(b) Give two Turing recognizable languages L_1 and L_2 such that $L_1 - L_2$ is not Turing recognizable.

Problem 2 Prove that the language

$$\text{HALT-ON-BLANK-TAPE}_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ halts when started on the blank tape} \}$$

is not decidable.

Hint: Show that $\text{HALT}_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM, } w \text{ is a string over } M\text{'s input alphabet, and } M \text{ halts on } w \}$ reduces to $\text{HALT-ON-BLANK-TAPE}_{TM}$, i.e., show that we can construct for each pair $\langle M, w \rangle$ a Turing machine that halts on the blank tape if and only if M halts on w .

Problem 3 Prove that the language

$$\text{ALWAYS-HALT}_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ halts on all inputs} \}$$

is not decidable.

Hint: Reduce $\text{HALT-ON-BLANK-TAPE}_{TM}$ to ALWAYS-HALT_{TM} .

Problem 4 Prove that the language

$$\text{FINITE}_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is finite} \}$$

is not decidable.

Problem 5 Problem 5.12, page 195 of *Sipser*.