

CS 3500 Section A
Sample Mid-Term Exam

Problem 1 (20 points) Give the state diagram of a DFA accepting the language specified by the regular expression $(0 \cup 10)^* 1^*$. (For half-credit you may give the diagram of an NFA with ε -transitions.)

Problem 2 (10 points) Give a regular expression for the language $\{x \in \{0, 1\}^* \mid x \text{ contains an odd number of 0's}\}$.

Problem 3 (20 points) Prove that if an NFA with k states accepts any string at all, then it accepts a string of length $k - 1$ or less.

Problem 4 (20 points) Prove that the language $\{0^{2^n} \mid n \geq 0\}$ is not context-free.

Problem 5 (20 points) Give a grammar in Chomsky normal form for the language $\{0^n 1^{2^n} \mid n \geq 0\}$. (For half-credit, give *any* context free grammar for this language.)

Problem 6 (20 points) Prove that the language $\text{USEFUL_STATE}_{TM} = \{\langle M, q \rangle \mid M \text{ is a TM, } q \text{ is a state of } M, \text{ and there exists a string } w \text{ such that state } q \text{ is reached during the computation of } M \text{ on input } w\}$ is undecidable.

Problem 7 (30 points) Which of the following languages are undecidable, and which are not? Give brief justification.

1. $\{\langle M \rangle \mid M \text{ has at least 100 states}\}$
2. $\{\langle M \rangle \mid M \text{ makes at least 100 transitions on input } \varepsilon\}$
3. $\{\langle M \rangle \mid M \text{ visits at least 100 distinct states on input } \varepsilon\}$
4. $\{\langle M \rangle \mid M \text{ makes at least 100 transitions on all inputs}\}$
5. $\{\langle M \rangle \mid M \text{ accepts at least 100 strings}\}$
6. $\{\langle M \rangle \mid M \text{ accepts some string of length at least 100}\}$