Bezier Patches

1 Definition

One can think of Bezier patches as Bezier curves of Bezier curves. To define a bi-cubic patch (which uses cubic Bezier curves), 16 points arranged in a 4x4 array are needed:

$$\begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{bmatrix}.$$

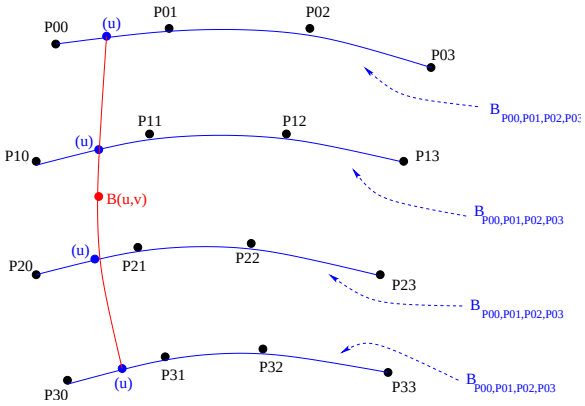

Figure 1: Recipe for a B-spline patch.

The points of the patch are parametrized by two parameters (say, u and v). Here's how to compuite the point $\bar{B}(u,v)$ (the point on the patch corresponding to parameter values u and v). First evaluate the four Bezier curves whose control points are rows of the matrix at u (Figure 1). The resulting four points can be used to define another Bezier curve, which is evaluated at the second parameter, v, leading to $\bar{B}(u,v)$. Using the notation from our first Bezier curve lecture (control points as subscripts), we can say that

$$\bar{B}(u,v) = B_{B_{P_{00}P_{01}P_{02}P_{03}}(u),B_{P_{10}P_{11}P_{12}P_{13}}(u),B_{P_{20}P_{21}P_{22}P_{23}}(u),B_{P_{30}P_{31}P_{32}P_{33}}(u)}(v).$$

The Bezier Patch is formed by all points of the form $\bar{B}(u,v)$ with u and v between 0 and 1. One can also say that it is what is swept by the moving Bezier curve controlled by the points $B_{P_{00}P_{01}P_{02}P_{03}}(u)$, $B_{P_{10}P_{11}P_{12}P_{13}}(u)$, $B_{P_{20}P_{21}P_{22}P_{23}}(u)$, $B_{P_{30}P_{31}P_{32}P_{33}}(u)$.

Our construction first builds curves controlled by rows of the matrix, but one could equally well start with four curves controlled by the columns, evaluate them at a parameter value, form the curve controlled by the resulting points

and evaluate it at the second parameter value. It turns out that this would lead to an identical patch.

Also, notice that this procedure works for bezier curves of any degree. Moreover, the 'row' degree does not have to be the same as the 'column' degree: our construction makes sense even for non-square matrices. Even more generally, it does not really matter how the curves are constructed. In particular, Bezier curves can be replaced by B-splines, four point rule curves or just anything, leading to other kinds of patches.

2 Drawing

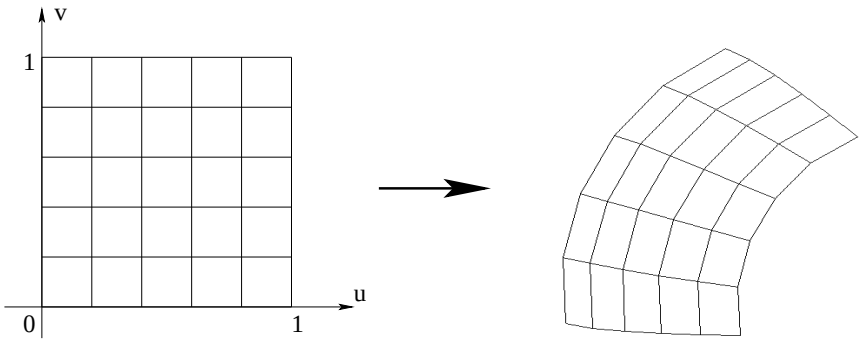

Figure 2: Drawing a patch.

Bezier patches can be drawn by subdivision, but the standard way offered by OpenGL is by evaluating the patch at a rectangular array of knots and joining the resulting points with quads. This is a lot similar to the way we drew spheres and tori. The number of knots controls the approximation accuracy (but, the more knots the more expensive the calculations). The efficient evaluation technique is the same as discussed in the context of polynomial curves. An important point to make is that *the number of knots in not in any way linked to the degree of the patch.* Both can be manipulated independently.