

Linear interpolation

‘Real’ functions are rarely given by an explicit formula allowing to evaluate them anywhere. More frequently, only ‘sample’ values of the function are given at certain points and, in order to estimate the value at some place which is not a sample, we need to somehow combine the available information. This is the goal of interpolation. By doing interpolation one can, for example, build a complete elevation map of a terrain when only an array of height values is given (this is what happens in practice: height cannot be measured everywhere; no matter how you do it, you end up with only finite number of measurements). We used interpolation to estimate the color over a triangle when colors at vertices are given, or normals/depths over the triangle from normals/depths at the vertices.

Linear interpolation from vertices of a triangle

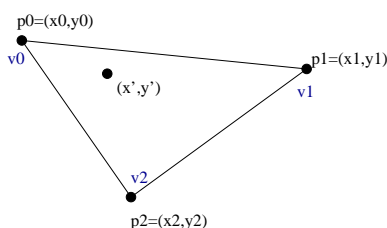


Figure 1: Linear interpolation from triangle’s vertices: notation.

Here is how the variant of linear interpolation we used works. Let’s say that we have a triangle as shown in Figure 1. It has vertices $p_0 = (x_0, y_0)$, $p_1 = (x_1, y_1)$, $p_2 = (x_2, y_2)$. At each vertex we have a value, let’s say that they are v_0 , v_1 and v_2 . There is exactly one linear function which takes the value of v_0 at p_0 , v_1 at p_1 and v_2 at p_2 (We can prove that by ‘embedding’ our triangle and the values at vertices in 3D: say that the triangle lies in the ground plane; lift p_0 to height v_0 , p_1 to height v_1 and p_2 to height v_2 . This yields 3 points in 3-space. Now, find the plane passing through the three points. This plane is the graph of the linear function we are looking for). Here is how to compute it algebraically.

We seek a function of the form $f(x, y) = Ax + By + C$ (A, B, C to be determined). The requirement that the values at the vertices are v_0 , v_1 and v_2 leads to three linear equations

$$Ax_0 + By_0 + C = v_0$$

$$Ax_1 + By_1 + C = v_1$$

$$Ax_2 + By_2 + C = v_2.$$

We can solve them, obtaining A, B and C . So, for example, to compute the interpolated value at the point $p' = (x', y')$ we need to evaluate $f(p') = f(x', y') = A * x' + B * y' + C$.