Practice problems[1]

1. The vertices of a triangle project to $(0,0)$, $(1,3)$ and $(3,1)$. Let's say that the depth values (computed using the perspective transformation matrix) at these vertices are 1, 3 and 4 (respectively). What is the depth of the point on that triangle which projects to $(1,1)$?
2. What is the triangle with vertices $(0,0)$, $(0,1)$ and $(1,0)$ mapped to by the transformation having the following matrix in the homogenous coordinates

$$\begin{bmatrix} 3 & 7 & 53 \\ 3 & 21 & 70 \\ 0 & 0 & 1 \end{bmatrix}$$

3. For the implicit surface $x^2 - y^2 - 12 = 0$ compute:

a. The closest intersection point of the ray starting at the origin and in the direction of $(2,1)$ (if exists)

b. A perpendicular vector to the surface at the intersection point (unless you figure out there is no intersection in a.)

4. (probably a little bit harder) We figured out that the polygon resulting from clipping a triangle against a rectangular viewport cannot have more than 7 vertices. Show that, in 3D, there is a triangle which clips (against a cube) to a polygon with 8 edges.
5. Let $B(t)$ be the 2D Bezier curve defined by four control points $(0,0)$, $(1,2)$, $(2,1)$ and $(3,0)$.

a. Give a formula for $B(t)$.

b. Compute the tangent vectors at the starting point ($t = 0$) and at the end-point ($t = 1$) by computing the derivatives at these parameter values; check that they are the same as the vectors joining the first and last two of the control points up to a scaling factor (this actually verifies that those vectors are really tangent to the curve – something we claimed in class).

[1] DISCLAIMER: This is by no means a complete study guide of any sort. I just sat down and wrote up a few questions which either you provided for me or which just somehow came to my mind. A question appearing here does not mean that the same or even a similar one will appear on the test. Also, a topic not being covered here does not mean that it won't be covered on Monday. Enjoy!

Some solutions

1. Use linear interpolation; Answer: 2.25

2. triangle with vertices (53,70), (60,91) and (56,73)

3. a. (4,2), b. (2,-1) or proportional

4. Try to modify the planar example where the triangle clipped to a 7-gon...

5. a. $B(t) = (3(1-t)^2t + 6(1-t)t^2 + 3t^3, 6(1-t)^2t + 3(1-t)t^2)$, b. ... not much to say here ....