Let $x' = x - \bar{x}$, and $y' = y - \bar{y}$. Setting the derivatives of E with respect to a', b', and c' to zero, and using the fact that

$$\sum x' = \sum (x - \bar{x}) = 0 \qquad\qquad \sum y' = \sum (y - \bar{y}) = 0$$

we get:

$$\begin{bmatrix} N & 0 & 0 \\ 0 & \sum x'^2 & \sum x'y' \\ 0 & \sum x'y' & \sum y'^2 \end{bmatrix} \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} \begin{bmatrix} \sum f \\ \sum x' f(x,y) \\ \sum y' f(x,y) \end{bmatrix} \tag{19}$$

In this case we get a' directly, and need to invert only a 2×2 matrix to get b' and c'. In fact, if the region R is rectangular (as is the case while we work with the quad-tree), $\sum x'y' = 0$, so that the matrix is diagonal, and inversion is trivial.

We can get the parameters a, b, and c directly from the parameters a', b', and c' by noting that:

$$a' + b'(x - \bar{x}) + c'(y - \bar{y}) = (a' - b'\bar{x} - c'\bar{y}) + b'x + c'y$$

and therefore concluding:

$$\begin{aligned} a &= a' - b'\bar{x} - c'\bar{y} \\ b &= b' \\ c &= c' \end{aligned}$$

The error of the fit can also be easily computed. Let

$$\hat{f}(x,y) = (a + bx + cy)$$

Then

$$\begin{aligned} E &= \sum (f(x,y) - \hat{f}(x,y))^2 \\ &= \sum f(x,y)(f(x,y) - \hat{f}(x,y)) - \sum \hat{f}(x,y)(f(x,y) - \hat{f}(x,y)) \end{aligned}$$

Using the well known orthogonality principle the second term is zero. Therefore:

$$\begin{aligned} E &= \sum f^2(x,y) - \sum f(x,y)\hat{f}(x,y) \\ &= \sum f^2(x,y) - a\sum f(x,y) - b\sum x f(x,y) - c\sum y f(x,y) \end{aligned} \tag{20}$$

It should be noted that the last three summations in this equation are also used in the computation of the parameters a, b, and c, and so do not need to be recomputed.

Plane-fitting of the flow fields in the split and merge algorithm is done independently for the x and y components giving a total of six parameters a_x, b_x, c_x, a_y, b_y, and c_y. The total error in the fit is taken to be the sum of the errors in the fits of the x and y components.

A final observation is that if along with each region, R_i, we keep the summations involved in the computation of its plane-fit parameters, then computing the parameters for a region $R_i \bigcup R_j$ is a relatively simple operation. We simply need to add up the individual sums from the two regions to get the corresponding summations for the merged region. Thus the merging operations used in