

CS 1050 H: Understanding and Constructing Proofs

Homework 6

Due Thursday, April 17, 2003

1. Let A_1, A_2, \dots be countably many sets, where each of A_1, A_2, \dots is a countable set. Show that $A_1 \cup A_2 \cup \dots$ is also a countable set.
2. Let $S = \{a + ib \mid a, b \in \mathbb{Z}\}$, where i is the square root of -1 . Is S countable or uncountable? Give a proof.
3. Construct four 3-sided dice with faces numbered from 1 to 12, each number used exactly once, say A, B, C, D , such that $A > B$, $B > C$, $C > D$ and $D > A$, where $\alpha > \beta$ means that dice α beats β more than half the time on random tosses.
4. A normal dice is six sided, numbered from 1 to 6. Construct three loaded normal dice, A, B, C , such that $A > B$, $B > C$ and $C > A$.
5. Consider a biased coin with $P(H) = p$ and $P(T) = 1 - p$, where $0 < p < 1$. Consider the experiment of tossing it a pair of times, repeating if the outcome is HH or TT and stopping if the outcome is HT or TH . If the total number of tosses before stopping is $2k$, then let us say that the experiment took k steps. Let H_f be the event that the experiment stops with HT and E be the event that the experiment takes an even number of steps. Compute $P(E)$, $P(H_f \mid E)$ and $P(E \mid H_f)$.

Note: Midterm 4 will be held on Thursday, April 17, 2003.