Problems 1 and 2 have been done in detail during the last review hour.

3:

- $8x = 1(mod 13)$: Using Euclid's algorithm for GCD, $8 * 5 - 13 * 3 = 1$. Therefore $x = 5(mod 13)$.
- $8x = 4(mod 13)$: $x = 5 * 4(mod 13)$, ie, $x = 7(mod 13)$.

4: $(m, n) = d)$. If a solution to $mx = a(mod n)$ exists, then $mx - a = ny$ for some y. That is $a = mx - ny$. Since d divides $mx - ny$, d divides a. For the second part, let d divide a. Let c be a solution to $\frac{m}{d}x = \frac{a}{d}(mod \frac{n}{d}$. Then $x = c + kd(mod n)$, $k = 0, 1, \ldots d - 1$ are solutions to $mx = a(mod n)$ (verify by substitution).

5: $am + bn = d$. Therefore, $a\frac{m}{d} + b\frac{n}{d} = 1$. Therefore $\frac{m}{d}$ and $\frac{n}{d}$ are coprime.

$am + bn = d = a'm + b'n$, ie, $(a - a')m = (b' - b)n$, ie, $(a - a')\frac{m}{d} = (b' - b)\frac{n}{d}$. Since $\frac{m}{d}$ and $\frac{n}{d}$ are coprime, $\frac{m}{d}$ divides $b' - b$. Let $k = (b' - b)/\frac{m}{d}$. k is an integer. Then $a - a' = k\frac{n}{d}$.