1: The number of arrangements of the word SOCIALOGICAL are $\dfrac{12!}{3!2!2!2!}$.

To count the number of words in which A and G are together, we can replace A and G by a single symbol (say @) and count the number of ways in which the new word can be rearranged, and then miltiply by the number of ways A and G can be arranged among themselves. Therefore, the number of words in which A and G are together is $\frac{11!}{3!2!2!2!} * 2$. The number of arrangements in which the vowels are together are $\frac{7!}{2!2!} * \frac{6!}{3!2!}$.

2: We can split this into 3 cases:

- Integers containing two 3s and two 7s: $\frac{4!}{2!2!}$.
- Integers containing two 3s and <= one 7 (so we have a choice from 1, 7 and 8 for the remaining two places): $\binom{3}{2}\frac{4!}{2!}$.
- Integers containing two 7s and <= one 3 (same as above): $\binom{3}{2}\frac{4!}{2!}$.
- Integers containing one 7 and one 3: 4!.

Add all the above numbers to get the final answer.

3: Replace the two persons who want to sit together by a dummy. Number of arrangements possible $= (6-1)! = 5!$. Now the two persons who want to sit together have 2 possible seating arrangements among themselves at the table. Therefore the total number of possible arrangements at the table $= 2 \times 5!$.

4: Number of distinct arrangements of the word UNUSUALLY $= \frac{9!}{3!2!}$
In $\frac{7!}{2!}$ of these, the Us are together.
In $\frac{8!}{3!}$ of these, the Ls are together. Therefore, in $\frac{9!}{3!2!} - \frac{8!}{3!}$ of these, the Ls are not together.

5a: $\binom{13}{2}$: We have 13 choices from which to pick 2 values.

5b: $6 * 4^8$: There are 6 possible values for the largest card. Once we make that choice, we have 4 possibilities (heart, spade, diamond, club) for each of the 8 positions.

5c: $\binom{13}{4}\binom{4}{2}^4$: We have 13 choices from which to pick the 4 values. We have 4 coices for the 2 card of each value.