CS3500B Homework 1 Solutions

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1.9

Given an NFA N_1, we construct N as follows. Add a new state q to N_1; this will be the new accept state. Add ϵ transitions from all the old accept states to q, and let q be the only accept state. Now we must show that $L(N) = L(N_1)$. Let $w \in L(N_1)$. Then $w \in L(N)$ because there must be a path for w through N_1 that ends in an accept state, so in N w just follows the ϵ transition to the new accept state. Now let $\hat{w} \notin L(N_1)$. Then there is no way for N to accept $\hat{w}$, because every path for $\hat{w}$ through N_1 ends in a non-accept state, and the only way to get to the accept state in N is from an accept state in N_1 with no input left. Thus N and N_1 accept the same strings, and so $L(N) = L(N_1)$.

5 1.10

a. Let M be the original DFA and let M' be obtained by swapping the accept and non-accept states of M. We wish to show that $L(M')$ is the complement of $L(M)$. Let $w \in L(M)$. When w is run through M, there is a unique path that it takes from the start state to an accept state. w follows the exact same path in M', but the path goes from the start state to a non-accept state, since we swapped the accept and non-accept states. Therefore $w \notin L(M')$. Conversely, let $\hat{w} \notin L(M)$. When $\hat{w}$ is run through M, there is a unique path from the start state to a non-accept state. But this path in M' ends at an accept state, so $\hat{w} \in L(M')$. We have shown that everything in $L(M)$ is not in $L(M')$ and that everything not in $L(M)$ is in $L(M')$. In other words, $L(M) \subseteq \overline{L(M')} \Rightarrow \overline{L(M)} \supseteq L(M')$ and $\overline{L(M)} \subseteq L(M')$, and thus $\overline{L(M)} = L(M')$, what we wanted to prove. From this we can conclude that the class of regular languages is closed under complement as follows. Let L be a regular language. Then there is a DFA that accepts L. Using the construction above, we can form a DFA that accepts $\overline{L}$. From this we know that $\overline{L}$ is regular. Thus the class of regular languages is closed under complement.

b. Notice in the figure that M is an NFA that accepts only 0; and M', which we obtained from M by swapping the start and accept states, accepts only ϵ. Clearly $\{\epsilon\}$ is not the complement of $\{0\}$, so this construction cannot be used with NFAs to demonstrate closure of the class of regular languages under complement. However, we do not conclude that the class of languages accepted by NFAs is not closed under complement. By Theorem 1.19, we know that the class of languages accepted by NFAs is the same as the class of regular languages, and we just proved above that this class is closed under complement.

6 problem 5

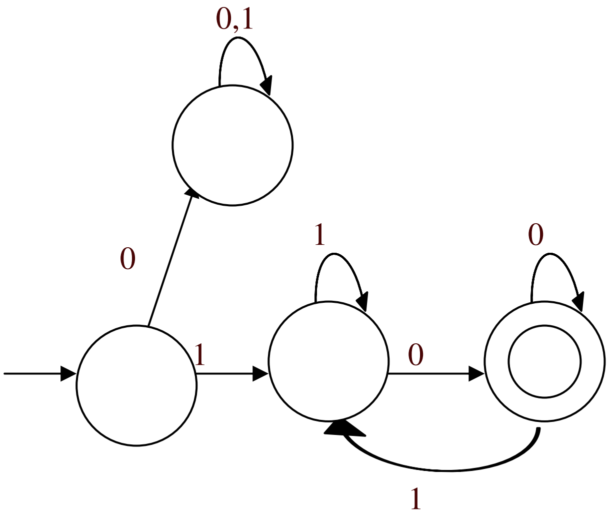
Assume that L_1 and L_2 are regular. We know that regular languages are closed under union and complement. So $L_3 = \bar{L_1} \cup \bar{L_2}$ is regular and so is $\bar{L_3} = L_2 \cap L_1$.

5 problem 6

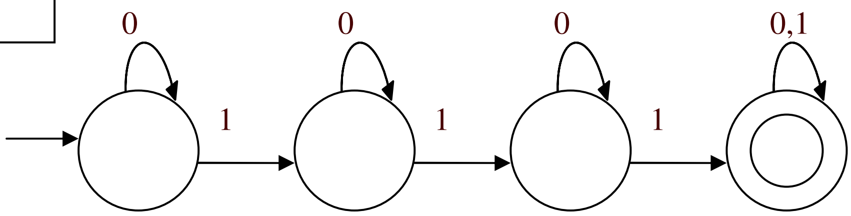
a) $1\Sigma^*0$

b) $\Sigma^*1\Sigma^*1\Sigma^*1\Sigma^*$
c) $\Sigma^*0\Sigma^*1\Sigma^*0\Sigma^*1\Sigma^*$
d) $\Sigma^*\Sigma^*\Sigma^*1\Sigma^*$
e) $(0\Sigma(\Sigma\Sigma)^*) \cup (1(\Sigma\Sigma)^*)$

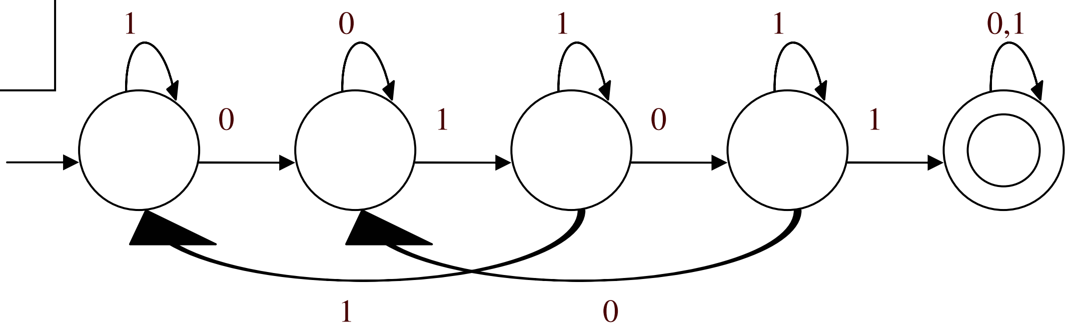
1.4a



1.4b



1.4c



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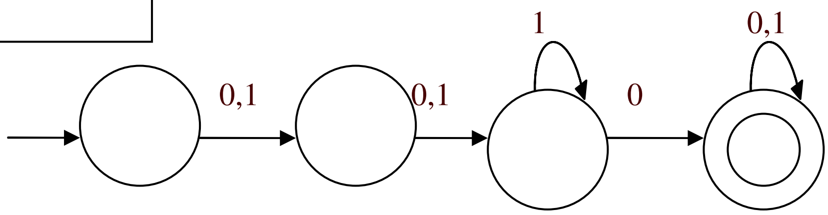

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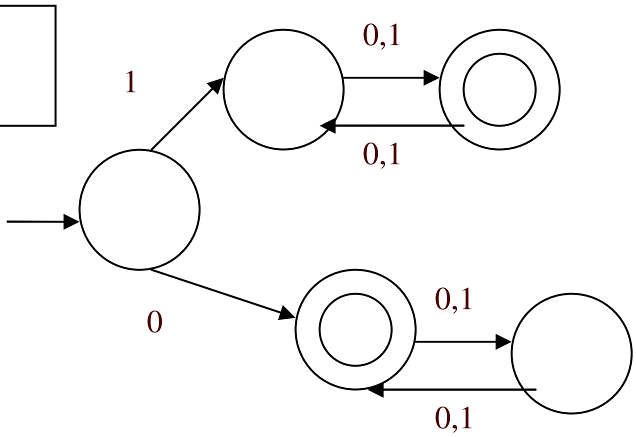
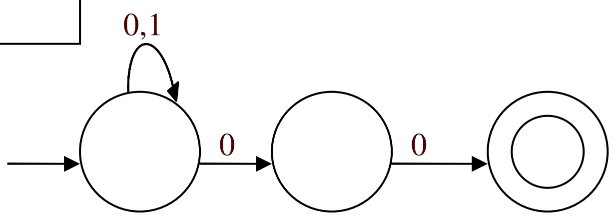


1.5a



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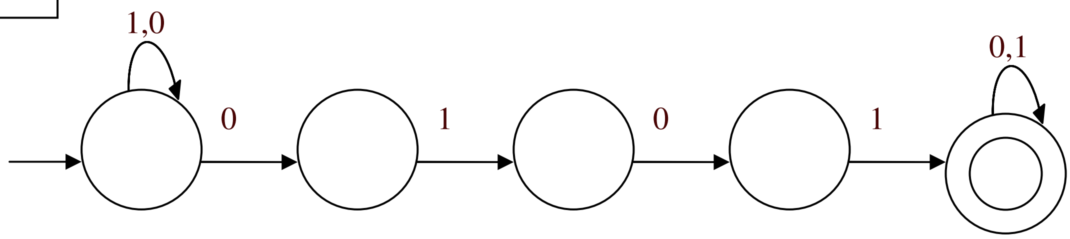

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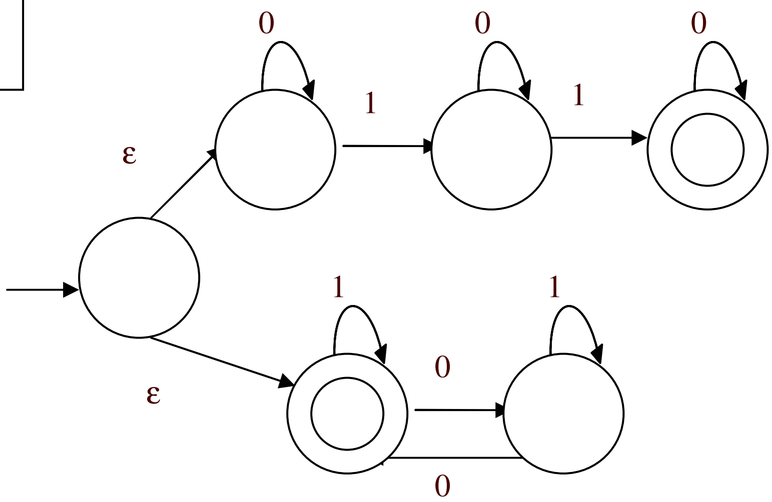

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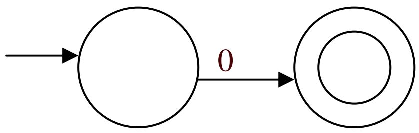

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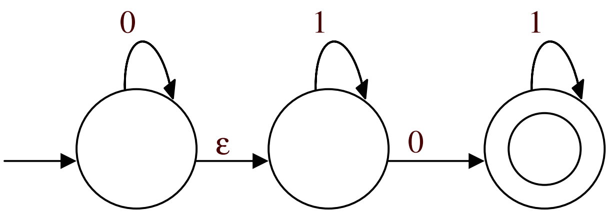

1.12a