

CS 3500: Introduction to the Theory of Computation

Problem Set 3

Problem 1 [40 Points]

For each of the following languages, determine whether it is *regular*, and prove your answer.

1. $L_1 = \{w : w \text{ does not have an occurrence of } 010 \text{ and } |w| \text{ is even}\}.$
2. $L_2 = \{xy : |x| = 3n + 2 \text{ for some } n \geq 0 \text{ and } y \in L(0^*1^*)\}.$
3. $L_3 = \{0^l1^m0^n : m = l + n\}.$
4. $L_4 = \{0^l1^m0^n : m \neq l + n\}.$

Problem 2 [40 Points]

For each of the following languages, determine whether it is *context-free*, and prove your answer.

1. $L_5 = \{a^i b^i c^j : j > i\}.$
2. $L_6 = \{a^i b^j a^k b^l : i + k = j + l\}.$
3. $L_7 = \{a^i b^j a^i b^j : i \geq 1, j \geq 1\}.$
4. $L_8 = \{a^i b^j c^k : i \neq j \text{ or } i \neq k\}.$

Problem 3 [10 Points]

Give an implementation-level description of a (multi-tape) Turing machine that adds two given integers in the binary representation.

Problem 4 [10 Points]

Exercise 4.2 on page 169 of Sipser.

Problem 5 [10 Points]

Exercise 4.3 on page 169 of Sipser.

Problem 5 [10 Points]

Exercise 4.5 on page 169 of Sipser.