

# CS 3500: Introduction to the Theory of Computation

## Problem Set 7

Due 4pm, July 25, 2003. NO LATE DAYS ALLOWED.

### Problem 1 [30 Points]

- (a) Problem 16-1 part (a) on page 402 of CLRS.
- (b) Problem 16-1 part (c) on page 402 of CLRS.
- (c) Problem 16-1 part (d) on page 402 of CLRS. (Hint: You may find dynamic programming helpful.)

### Problem 2 [30 Points]

Problem 25-1 on page 641 of CLRS.

### Problem 3 [20 Points]

- (a) Define the language

$\text{LPATH} = \{\langle G, s, t, k \rangle : G \text{ is a graph that contains a path of length at least } k \text{ from vertex } s \text{ to vertex } t.\}$

By reduction from HAMILTONIAN-PATH, prove that LPATH is NP-Complete.

- (b) Let  $G = (V, E)$  an undirected graph. A subset  $S \subset V$  of vertices is an *independent set* if  $\forall u, v \in S, (u, v) \notin E$ , in other words, there is no edge between any two vertices in  $S$ . Define the language

$\text{INDEPENDENT-SET} = \{\langle G, k \rangle : G \text{ is a graph that contains an independent set } k.\}$

By reduction from CLIQUE, prove that INDEPENDENT-SET is NP-Complete.

### Problem 4 [20 Points]

By reduction from 3SAT, prove that the following problem is NP-Hard.

**Input:** An  $m \times n$  matrix  $A$  in which each entry is an integer, and a column vector  $b$  of  $m$  integers.

**Output:** Whether there is a column vector  $x$  of  $n$  integers such that  $Ax \geq b$ .