Ray-triangle intersection:

1. (-2,2,5)

2. no intersection

3. no intersection

Ray-sphere intersection:

1. $(-4 + \frac{22+\sqrt{4908}}{28}, -2 + 2\frac{22+\sqrt{4908}}{28}, -1 + 3\frac{22+\sqrt{4908}}{28})$

2. $(-3 - \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, 2 - \frac{3}{\sqrt{14}})$

3. no intersection

4. no intersection

Solution of ray-triangle problem 1.

$$a_1\vec{a}_2 = [-3, -5, -2]$$

$$a_1\vec{a}_3 = [-2, -5, -3]$$

$$\vec{n} = \left[\left| \begin{matrix} -5 & -2 \\ -5 & -3 \end{matrix} \right|, \left| \begin{matrix} -2 & -3 \\ -3 & -2 \end{matrix} \right|, \left| \begin{matrix} -3 & -5 \\ -2 & -5 \end{matrix} \right| \right] = [5, -5, 5]$$

$$a_1\vec{o} = [-4, -8, -8]$$

$$t = -\frac{a_1\vec{o} \cdot \vec{n}}{\vec{d} \cdot \vec{n}} = -\frac{-4 * 5 + 8 * 5 - 8 * 5}{5 - 2 * 5 + 3 * 5} = 2$$

$$p = (-4 + 2 * 1, -2 + 2 * 2, -1 + 2 * 3) = (-2, 2, 5)$$

Now we test whether p is inside the triangle using the cross product method:

$$p\vec{a}_1 = [2, 4, 2]$$

$$p\vec{a}_2 = [-1, -1, 0]$$

$$p\vec{a}_3 = [0, -1, -1].$$

The x-coordinates of the cross product of interest are:

$$(p\vec{a}_1 \times p\vec{a}_2)_x = \left| \begin{matrix} 4 & 2 \\ -1 & 0 \end{matrix} \right| = 2$$

$$(p\vec{a}_2 \times p\vec{a}_3)_x = \left| \begin{matrix} -1 & 0 \\ -1 & -1 \end{matrix} \right| = 1$$

$$(p\vec{a}_3 \times p\vec{a}_1)_x = \left| \begin{matrix} -1 & -1 \\ 4 & 2 \end{matrix} \right| = 2$$

Since all of them are positive, p is inside the triangle and therefore the ray intersects the triangle at $(-2, 2, 5)$.

Solution of ray-triangle problem 2.

$$\vec{a_1a_2} = [1, 0, -1]$$

$$\vec{a_1a_3} = [3, 5, 2]$$

$$\vec{n} = \left[\left| \begin{array}{cc} 0 & -1 \\ 5 & 2 \end{array} \right|, \left| \begin{array}{cc} -1 & 1 \\ 2 & 3 \end{array} \right|, \left| \begin{array}{cc} 1 & 0 \\ 3 & 5 \end{array} \right| \right] = [5, -5, 5]$$

$$\vec{a_1o} = [-2, -3, -3]$$

$$t = -\frac{\vec{a_1o} \cdot \vec{n}}{\vec{d} \cdot \vec{n}} = -\frac{-2*5+3*5-3*5}{5-2*5+3*5} = 1$$

$$p = (-4+1*1, -2+1*2, -1+1*3) = (-3, 0, 2)$$

Now we test whether p is inside the triangle using the cross product method:

$$\vec{pa_1} = [-3, -5, -6]$$

$$\vec{pa_2} = [-2, -5, -7]$$

$$\vec{pa_3} = [0, 0, -4].$$

The x-coordinates of the cross product of interest are:

$$(\vec{pa_1} \times \vec{pa_2})_x = \left| \begin{array}{cc} -5 & -6 \\ -5 & -7 \end{array} \right| = 5$$

$$(\vec{pa_2} \times \vec{pa_3})_x = \left| \begin{array}{cc} -5 & -7 \\ 0 & -4 \end{array} \right| = 20$$

$$(\vec{pa_3} \times \vec{pa_1})_x = \left| \begin{array}{cc} 0 & 4 \\ -5 & -6 \end{array} \right| = -20$$

They are of different signs, so the intersectrion point is outside the triangle. The ray misses the triangle.

Solution of ray-triangle problem 3.

$$\vec{a_1 a_2} = [2,4,6]$$
$$\vec{a_1 a_3} = [1,0,0]$$
$$\vec{n} = \left[\begin{vmatrix} 4 & 6 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 6 & 2 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 4 \\ 1 & 0 \end{vmatrix} \right] = [0,6,-4]$$
$$\vec{a_1 o} = [-4,-2,-1]$$
$$t = -\frac{\vec{a_1 o} \cdot \vec{n}}{\vec{d} \cdot \vec{n}} = -\frac{-2*6+4}{2*6-3*4} = \text{....Ooops... division by zero}$$

In fact, what is happening is that the ray is parallel to the plane (the normal of the plane is orthogonal to the direction vector). In this case, the ray is not contained in the triangle's plane (otherwise, the numerator in the equation for t would also be zero!) so there is no intersection.

In practice, whenever the ray is parallel to the triangles plane we can report no intersection. Even if we are wrong (the ray is in the triangle's plane and in fact intersects it) the consequences for the ray traced image are not that important.

Solution of ray-sphere problem 1.

$$A = 1^2 + 2^2 + 3^2 = 14$$
$$\vec{co} = [-4, -2, -1]$$
$$B = 2(-4 * 1 - 2 * 2 - 1 * 3) = -22$$
$$C = 4 * 4 + 2 * 2 + 1 * 1 - 10^2 = -79$$
$$\Delta = B^2 - 4AC = 4908$$
$$t = \frac{-B \pm \sqrt{\Delta}}{2A} = \frac{22 \pm \sqrt{4908}}{28}$$

The solution with a '-' is negative and therefore does not interest us (note that since the solutions have different signs, the origin is inside the sphere). The closest intersection happens at

$$t = \frac{-B \pm \sqrt{\Delta}}{2A} = \frac{22 + \sqrt{4908}}{28}.$$

The coordinhates of the intersection point are

$$\left(-4 + \frac{22 + \sqrt{4908}}{28}, -2 + 2\frac{22 + \sqrt{4908}}{28}, -1 + 3\frac{22 + \sqrt{4908}}{28}\right)$$

Solution of ray-sphere problem 2.

$$A = 1^2 + 2^2 + 3^2 = 14$$

$$\vec{co} = [-1, -2, -3]$$

$$B = 2(-1 * 1 - 2 * 2 - 3 * 3) = -28$$

$$C = 3 * 3 + 2 * 2 + 1 * 1 - 1^2 = 13$$

$$\Delta = B^2 - 4AC = 56$$

$$t = \frac{-B \pm \sqrt{\Delta}}{2A} = \frac{28 \pm \sqrt{56}}{28}$$

In this case, both solutions are positive (which means that the ray intersects the sphere at two points). Therefore, we select the smaller t:

$$t = \frac{-B \pm \sqrt{\Delta}}{2A} = \frac{28 - \sqrt{56}}{28} = 1 - \frac{1}{\sqrt{14}}.$$

The coordinates of the intersection point are

$$\left(-4 + 1 - \frac{1}{\sqrt{14}}, -2 + 2(1 - \frac{1}{\sqrt{14}}), -1 + 3(1 - \frac{1}{\sqrt{14}})\right)$$

Solution of ray-sphere problem 3.

$$A = 1^2 + 2^2 + 3^2 = 14$$

$$\vec{co} = [1, 4, 6]$$

$$B = 2(1 * 1 + 4 * 2 + 6 * 3) = 54$$

$$C = 1^2 + 4^2 + 6^2 - 2^2 = 49$$

$$\Delta = B^2 - 4AC = 172$$

$$t = \frac{-B \pm \sqrt{\Delta}}{2A} = \frac{-54 \pm \sqrt{172}}{28}$$

Both solutions are negative. Therefore, there is no intersection.

Solution of ray-sphere problem 4.

$$A = 1^2 + 2^2 + 3^2 = 14$$

$$\vec{co} = [-4, -2, 0]$$

$$B = 2(-4 * 1 - 2 * 2 + 0 * 3) = -16$$

$$C = 4 * 4 + 2 * 2 - 3^2 = 11$$

$$\Delta = B^2 - 4AC = -360 < 0$$

Therefore, the ray misses the sphere.