# Physical Limits of Portable Power Storage ROUGH DRAFT! DO NOT DISTRIBUTE!

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## 1 Introduction

## 2 Superflywheel

A flywheel is a type of electromechanical energy storage system. Energy is stored as rotational kinetic energy by spinning a disk of material very quickly. Energy loss is minimized by keeping frictional losses (both from the bearings at the axle and air resistance) low. Flywheels have been under active research for the last decade as an energy storage medium for electric and low-pollution/high efficiency vehicles. Conversion from electrical to mechanical energy and *vice versa* is quite efficient using permanent magnets on the flywheel and a coil to generate current. Efficiency values greater than 90% were reported. Research in the field has also produced extremely low friction bearings using superconducting magnets. These magnetic bearings and Active Magnetic Bearings (which use an active controller to avoid contact) actually support the flywheel using magnetic forces so that there is no contact or friction and no lubricants involved. In addition, air resistance can be virtually eliminated by housing the flywheel in a vacuum bottle. This bottle can also serve as a shield in case of catastrophic failure. Thus, the only limit to the speed at which the wheel can be spun is the ultimate strength of the materials of which it is made, which will be the focus of this analysis. Finally, it should be noted that experimentally it has been shown that when wheels made of carbon fibre composites breakdown, they turn to a fine dust

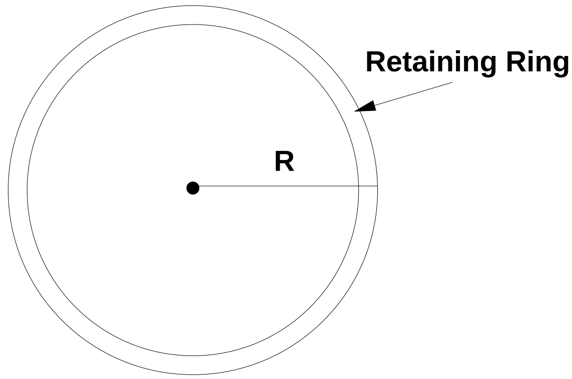


Figure 1: A simple flywheel design

rather than shrapnel, which means that a light Kevlar shield is enough to contain a breakdown.

## 2.1 Analysis of energy and breakdown stress

First recall that the rotational kinetic energy of a rigid body rotating around a single fixed axis is:

$$K = \frac{1}{2} I \omega^2 \tag{1}$$

where $I$ is the moment of inertia around that axis. The moment of inertia is defined as:

$$I = \int \rho R^2 dV \tag{2}$$

where $\rho$ is the density of the substance and $R$ is the distance from the axis of the differential volume element $dV$. For a simple flat disk which we will consider, this is simply:

$$I = \frac{1}{2} M R^2 \tag{3}$$

where $M$ is the mass of the disk. Hence,

$$K = \frac{1}{4} M R^2 \omega^2 \tag{4}$$

The energy goes as the square of the angular velocity and as the square of the radius, but only linearly with mass. Thus, we can expect that smaller flywheels will not be nearly as powerful as those boasted for vehicle use, unless we can increase the speed by a significant amount.

Assuming magnetic bearings and a vacuum, the limiting factor on the speed of the flywheel is the ultimate tensile strength (how much force it can take before deforming permanently or breaking apart) of the material on the outer rim, which experiences the largest centrifugal load. It can be shown that the stress $s$ on a thin retaining ring around the rim of the flywheel is equal to [Seely and Ensign, 1952]:

$$s = \rho R^2 \omega^2 = \rho v^2 \tag{5}$$

where a $\rho$ is the mass density of the rim material. If $\rho$ is in units of $\frac{kg}{m^3}$, then $s$ is in units of $\frac{N}{m^2}$, or Pascals $(Pa)$[1]. Notice that the stress is proportional to the rim velocity and the density. This implies that to maximize $v^2$, we desire a high strength, low-density material in a retaining rim around the flywheel to hold it together. In other words, given a maximum tensile strengh $s_{max}$, the maximum angular velocity of the flywheel is:

$$\omega_{max} = \sqrt{\frac{s_{max}}{\rho}} \frac{1}{R} \tag{6}$$

Combining equations, we notice that maximum energy capacity of a fixed size flywheel is proportionally constrained by the tensile strength of the rim by:

$$K_{max} = \frac{m \ s_{max}}{4\rho} \tag{7}$$

Thus, the maximum allowable energy storage in a flywheel of any size depends only on the strongest and lightest materials we can make. We can make linear improvements in energy storage by increasing mass, but the energy is independent of radius, which is slightly surprising!

Various high tensile strength carbon fibre composites exist and others are being developed. For example, IM7 carbon fibre has an ultimate tensile strength of 4.8 $GPa$and a density around 1.8 $\frac{g}{cm^3}$$(1.8 \times 10^3 \ \frac{kg}{m^3})$. More expensive carbon fibre composites under current development have a tensile

[1]To convert from psi (pounds per square inch) to Pa, multiply by $.6867 \times 10^4$

strength around 10 $GPa$. As a side note, several researchers are investiagting a theoretical substance known as a "buckytube", which is basically a tube made from buckminsterfullerene (buckyball). Claims of 100 times the strength of steel at a quarter the density are being made [?]. We make no claims about this substance, but if it existed, it could have a strength around 100 $GPa$.

We now analyze a fairly small flywheel of the sort that could fit in a wearable computer. A reasonable radius is $5cm$, which is about 5 inches in diameter. We can also assume it has some thickness, say $1cm$. The volume is then about $80cm^3$. We will assume that it weights $1kg$. The obvious first calculation is what kinds of spin speeds will be necessary in order to store a certain amount of energy. Solving Equation 4 for the angular velocity gives us:

$$\omega = \frac{2\sqrt{\frac{K}{M}}}{R} \tag{8}$$

For the wheel described above with a rim of IM7 carbon fibre material, we could reach a maximum spin speed of 32,700 $\frac{radians}{s}$or 5,200 $hz$. This is a maximum energy storage of 185 $W \cdot h$ . The more experimental, costly fibers (10 $GPa$) can sustain a maximum speed of 7,400 $hz$. Using the more exotic carbon fibre with an $s_{max}$ of 10 $GPa$gives us a maximum energy of:

$$K_{max} = \frac{(1kg)\ (1.0 \times 10^{10} \frac{N}{m^2})}{(4)\ (1.8 \times 10^3 \frac{kg}{m^3})} = 1.39MJ = 385W \cdot h \tag{9}$$

Since this is the ultimate limit, it is suggested that the wheel be spun up to 70% of its maximum value to avoid permanent deforming or breakdown. This also does not take into account a Kevlar shield in case it does fail.

## 2.2 Gyroscopic forces

Gyroscopic forces will make it difficult to turn a fast flywheel. In particular, the angular momentum of a flywheel will be:

$$\vec{L} = I\omega = \frac{1}{2}MR^2\omega \tag{10}$$

with direction along the flywheel axis. Also, since

$$\frac{d\vec{L}}{dt} = \tau = \vec{r} \times \vec{F} \tag{11}$$

we see that an external force applied to the flywheel will cause a change in the angular momentum perpendicular to the force. Rather that fully characterize the actual forces here, we note that the equation for the anuglar mometum $\vec{L}$ applies to the entire rigid system. Therefore, if two flywheels are connected by a rigid shaft, the sum angular momentum of the system will be zero, allowing us to turn it easily. A few issues apply here:

- The wheels need to be spun down and up simultaneously in order to avoid getting a non-zero net angular momentum.
- The actual viability of this method depends on the shear strength of the connecting shaft, which must be able to transfer the torque without snapping or bending.

# 3 Compressed air tanks

Another storage method is to use a tank of compressed air to power a small turbogenerator. A small turbine could be spun by the escaping gas in order to produce electricity. We will assume an adiabatic expansion of the gas, although this is not strictly true since there is heat transfer to the container, etc. In the case of adiabatic expansion, the escaping gas will be cooler than the ambient air since its intrinsic energy is being used to expand and power the turbine. As an additional bonus, we could use the cool air to cool our CPU.

For simplicity of calculation, we assume that the expansion is adiabatic and the gas ideal. The equation for adiabatic expansion is

$$pV^{\gamma} = [constant] \tag{12}$$

where $\gamma$ is 1.4 for air. Also recall that the work done by an expanding gas is:

$$\int_{V_1}^{V_2} p\ dV \tag{13}$$

This is an integral over volume, but the volume of the tank is fixed. The gas is, however, escaping into the environment, where it will finally take up some volume. Since Equation 12 must be true before and after the expansion to atmospheric pressure, the final volume at $1atm$ must be:

$$V_{atm} = V_{tank} \left(\frac{p_{tank}}{p_{atm}}\right)^{\gamma-1} \tag{14}$$

Since $p$ changes as we release the gas, we need an equation for it in Equation 13. Again using Equation 12, we see that

$$p = p_{tank} \left(\frac{V_{tank}}{V}\right)^{\gamma} \tag{15}$$

Plugging this into Equation 13 and integrating from $V_{tank}$ to $V_{atm}$ (as calculated in Equation 14) gives us a total available work of:

$$W = \frac{1}{\gamma - 1} p_{tank}\, V_{tank} \left[1 - \left(\frac{p_{atm}}{p_{tank}}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{16}$$

Air turbine and generator systems have a ballpark upper efficiency of 40% [**?**], with 20% probably being a more practical number for a small turbine. The limits are:

- Encumbrance of tank (volume and weight)
- Maximum pressure allowable in a tank before rupture

At a given pressure, volume and temperature, the ideal gas law will give us the mass of the gas:

$$p\, V = \left(\frac{m}{M}\right) R\, T \tag{17}$$

where $M$ is the molar weight of the gas in $kg$, $m$ is the gas total mass. For $T = 300 \deg K$, $V = 1liter = 10^{-3}m^3$, molar weight of air as $29g$, the mass as a function of pressure is:

$$m = 1.17 \frac{g}{atm} \tag{18}$$

Clearly this is negligible compared to tank weight. The maximum energy (work) available for a tank with rupture pressure $p_r$ is then:

$$W_{max} = (2.5)p_r V_t \left[ 1 - \left( \frac{1.0 \times 10^5 \frac{N}{m^2}}{p_r} \right)^{.286} \right] \tag{19}$$

with units of Joules if $p_{max}$ is in $\frac{N}{m^2}$ and $V_t$ is in $m^3$.

Portable compressed air systems are in common use today for powering paintball guns [Hoover, ]. Tanks are designed to mount on guns or be strapped onto one's back. They are usually made of reinforced aluminum and wrapped in fiberglass and have a working pressure of around 3000 psi (207 atm). A 68 $in^3$tank (about 1 liter) weighs around 2 pounds (1 kg) and is considered safe (can be shot and penetrated and not explode, only vent). Plugging this pressure for a one liter tank into Equation 19:

$$W_{max} = (2.5)(207 \times 10^5 \frac{N}{m^2})(1.0 \times 10^{-3} m^3)(1 - \left(\frac{1}{207}\right)^{.286}) \tag{20}$$

$$= 43,729J \tag{21}$$

$$= 12W \cdot h \tag{22}$$

gives a theoretical energy of (12 $W \cdot h$ ), times the theoretical efficiency of the turbine (40%) gives us 5.75 $W \cdot h$ . Since this is for a 1 liter, 1 kg tank, this value can be considered a volumetric or gravimetric energy density. This number must be taken with a grain of salt, however, since we ignore the weight and volume of the turbogenerator. Calculation of the limits on this is beyond the scope of this paper. This is not very good as compared with batteries.

Note also that these values are for a commercial product which can be bought for about $250 from a paintball supplier. Research on stronger low-density composites should increase this value.

# 4 Ultracapacitor

Another potential source of power storage is the capacitor. Currently there is a lot of interest in high energy density capacitors (commonly referred to as ultracapacitors or supercapacitors) for use in electric vehicles to store energy during braking and use it for acceleration. Various research labs are trying different approaches [Anonymous, ]. Projected energy densities for current

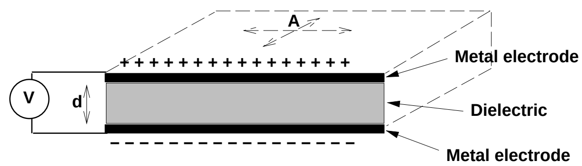


Figure 2: Parallel plate capacitor

devices range from 3–30 $\frac{W \cdot h}{kg}$, which is in the low range for batteries. Capacitors, however, have the distinct advantage that they can be be recharged incrementally and quickly, as opposed to batteries. Thus, capacitors would be much more useful in a hybrid system, such as using photocells during high illumination conditions and storing excess power.

Two types of capacitor are being studied:

- Parallel plate
- Double layer

We do an analysis for the parallel plate case and a more empirical state-of-the-art discussion of the less understood double layer capacitors.

## 4.1 Parallel plate

The first capacitor is the familiar solid-state metal-insulator-metal stack, as shown in Figure 2. The capacitance of such a system can be derived as:

$$C = \frac{A}{d}\, \epsilon_0\, \epsilon_r \tag{23}$$

where $A$ is the area of the electrode, $d$ is the plate separation, $\epsilon_0$ is the permittivity of free space and $\epsilon_r$ is the relative permittivity, or dielectric constant, of the dielectric between the plates. The energy stored in a capacitor is:

$$E = \frac{1}{2}\, C\, V^2 \tag{24}$$

where $V$ is the potential across the plates. We can increase the energy by increasing $C$ or $V$. We can increase $C$ by increasing the dielectric constant or be decreasing the electrode spacing $d$, but clearly this cannot be done arbitrarily. We can also increase $V$, but at some maximum field strength,

$E_b$, the dielectric will fail and conduct current, usually destructively. This is called *dielectric breakdown* and is caused by various methods, the theory of which is not very developed but some of which can be found in [Sze, 1981]. In general the value of $E_b$ must be calculated empirically. $E_b$ has units of $\frac{V}{m}$since it is an electric field magnitude.

For a spacing of $d$, we can store a maximum potential $V_{max}$ of $E_b\ d$. Combining Equations 23 and 24 and using this fact gives us the energy with respect to a given dielectric:

$$E = \frac{1}{2}\ A\ d\ \epsilon_0\ \epsilon_r\ E_b^2 \tag{25}$$

Fletcher *et al* [Fletcher *et al.*, 1996] quote Love's [Love, ] empirical characterization of maximizing energy density:

> Maximum energy storage is not obtained in high dielectric constant materials but in thos materials which display intermediate dielectric constant and highest ultimate breakdown voltage.

This is not surprising since the energy depends on the square of the breakdown voltage and only linearly on dielectric constant.

**thin film polymers** Thin film polymers are the main dielectric being considered for parallel plate ultracapacitors. Normal polymer (like polystyrene) low-frequency dielectric constants range from 2-8 and dielectric strengths are in the range 100–600 $\frac{kV}{cm}$, or 10–60 $\frac{V}{\mu m}$ [Anderson, 1989] for a 3$mm$ film. Other polymers such as polyethylene terephthalate (PET, used to make soft drink bottles) have an $E_b$ of about 6000 $\frac{kV}{cm}$, or about 600 $\frac{V}{\mu m}$, for a film 28 $\mu m$thick [Zheng *et al.*, 1996]. This can be increased by about 10–18% by laminating the polymers with polyvinylidene fluoride (PVDF), commonly used for its piezoelectric properties, giving a strength of 711 $\frac{V}{\mu m}$for a 28 $\mu m$film of PVDF laminated PET [Zheng *et al.*, 1996] (we'll call this PET+). The dielectric constant for this material was around 3.

Using these values, we calculate the volumetric energy density of a parallel plate capacitor using a PET+ dielectric, ignoring the volume of the electrodes for now. Since the volume of the dielectric is $A\ d$, we divide it out to give an energy density of the dielectric $U$ of:

$$U_d = \frac{1}{2}\ \epsilon_0\ \epsilon_r E_b^2 \frac{J}{m^3} \tag{26}$$

For PET+ this is

$$U_d = (.5)\ (8.854 \times 10^{-12} \frac{F}{m})\ (3)\ (711 \frac{V}{\mu m}\ 10^6 \frac{\mu m}{m})^2 = 6.7 \frac{MJ}{m^3} = 1861 \frac{W \cdot h}{m^3} \tag{27}$$

Most variations of polyethylene have a specific gravity close to water ($1\ \frac{g}{cm^3} = 1000\ \frac{kg}{m^3}$), which means that ignoring the weight of the electrodes and casing, the gravimetric energy density can be calculated simply since $m = \rho\, V$ where $\rho$ is the density in $\frac{kg}{m^3}$. We do this by dividing the volumetric energy density by the dielectric density:

$$\frac{1861 \frac{W \cdot h}{m^3}}{1000 \frac{kg}{m^3}} = 1.86 \frac{W \cdot h}{kg} \tag{28}$$

This is quite low, especially since it excludes electrode mass.

Doping polymers also can increase dielectric breakdown fields. Adding paraffin wax to polypropylene produced a 24 $\mu m$film with a dielectric constant of about 2 and a breakdown stregnth of about 315 $\frac{V}{\mu m}$ [William L. Wade *et al.*, 1993]. This gives a dielectric energy density $U_d$ of 122 $\frac{W \cdot h}{m^3}$, which is lower by an order of magnitude.

**ceramics** Ceramics are also under study as dielectrics. Recent ceramics have dielectric strengths of 100 $\frac{V}{\mu m}$ [Fletcher *et al.*, 1996], which is lower than polymers. The dielectric constants tend to be much larger than polymers, however, around 3000 [Systems, ]. Using these values, we get a $U_d$ of 133 $\frac{MJ}{m^3}$(37 $\frac{kW \cdot h}{m^3}$). Ceramics weigh more than polymers, though, around 8000 $\frac{kg}{m^3}$, so our dielectric gravimetric density about 4.6 $\frac{kW \cdot h}{kg}$, which is still lower than batteries, and again excludes the electrodes and packaging.

## 4.2 Double layer

The more common method for making high-density capacitors is double layer capacitance, the phenomenon where charge collects in the interface between two dissimilar materials [Ballard *et al.*, 1989], usually an electrolyte and conductor. By increasing the interface surface area, the charge storage is increased, and thus the capacitance. Energy densities of 20 $\frac{MJ}{m^3}$(5500 $\frac{W \cdot h}{m^3}$) for a sulfuric acid electrolyte and ceramic conductor (at 1.2 V, the breakdown voltage for sulfuric acid) were given [Ballard *et al.*, 1989]. If a solid electrolyte is used with higher breakdown voltage, values of 70 $\frac{MJ}{m^3}$(20,000 $\frac{W \cdot h}{m^3}$) were projected. Corrosive effects of the sulfuric acid on the ceramic must be taken into account, however. Also, these ultracapacitors were being developed for rail gun pulse applications, so they tended to have a high leakage since they wouldn't be used for long periods of time.

### 4.3 Electrode size limits

The electrode metal is often deposited onto either side of the dielectric in parallel plate capacitors. The thinner this can be made, the higher the total energy density can be. The limit on miniaturation of the electrode tends to be electromigration, which is the formation of gaps in the electrode due to the pressure of the current [Fraser, 1986]. This is expressed in terms of the current density ($\frac{A}{m^2}$). For Al it is about $10^10$ $\frac{A}{m^2}$, so a suggested safe value is $10^9$ $\frac{A}{m^2}$. This value deals with the cross-sectional area, which is the depth of the capacitor times the thickness of the electrode. This means that the following relation must hold between the current, electrode thickness $t$ and current collector width $w$:

$$\frac{I}{t\ w} \leq 10^9 \tag{29}$$

or

$$t \geq \frac{I}{10^9\ w} \tag{30}$$

Assume we want to draw 1 W at 5 V from the polymer capacitor described above. Since $P = I\ V$, the current drawn $I$ is .2 A. If our capacitor is 1 cm across at the current collector, its thickness would have to be:

$$t \geq \frac{.2}{(10^9)\ (10^-2)} = .02\mu m \tag{31}$$

which is negligible compared to the dielectric width, but will add some weight.

## 5 Fuel cells

Fuel cells, such as hydrogen-oxygen fuel cells, are commonly thought to combust the hydrogen and convert the thermal energy into electricity. Actually,

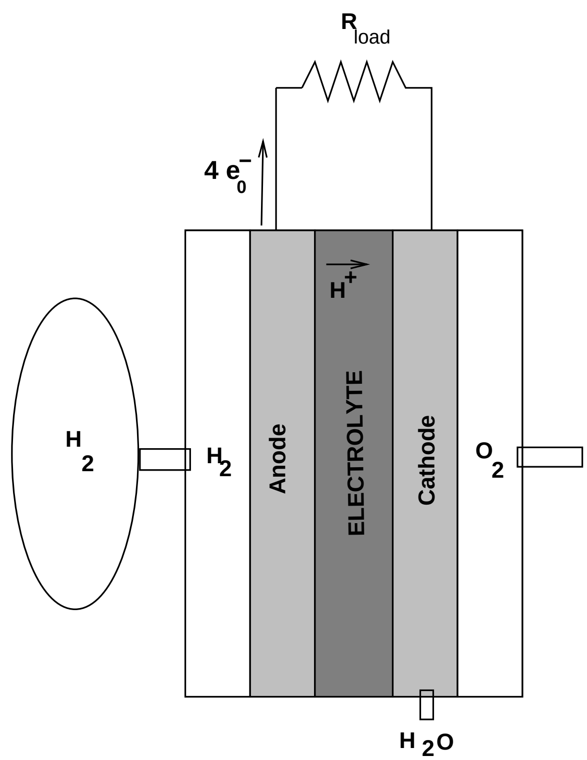


Figure 3: A hydrogen-oxygen fuel cell design

they are electrochemical devices that use an oxidation-reduction (redox) reaction to produce a useful current and some by-products. Thermal energy is often produced, but is not the main source of energy. We will focus on hydrogen fuel cells for this paper, since they are simple, clean and can be regenerative. We will also mention several other possibilities for other fuels in fuel cells.

## 5.1 Hydrogen-oxygen (air) fuel cells

Figure 3 depicts a simple fuel cell design as depicted in [Bockris and Srinivasan, 1959]. Hydrogen gas enters the gas chamber on the left, air enters the chamber on the right. The anode is a porous electrocatalyst that strips the electrons from the hydrogen (oxidization). The electrolyte is an ion-exchange medium, allowing only $H^+$ ions to cross and not H atoms. Likewise, it cannot allow $O^{2-}$ ions to flow the other way. A circuit is connected from anode to cathode allowing the electrons to reduce the oxygen on the porous cathode. The hydrogen ions join the oxygen ions to produce water and some thermal energy. The water drains out the bottom.

The anode equation is

$$H_2 \rightarrow 2\ H^+ \ + \ 2e_0^- \tag{32}$$

and the cathode equation is

$$O_2 + 4\ H^+ + 4e_0^- \rightarrow 2\ H_2O \tag{33}$$

for an overall reaction of

$$2\ H_2 + O_2 \rightarrow 2\ H_2O \tag{34}$$

The ideal available electrical work (assuming no losses by heat) from the electrons flowing through the circuit is:

$$W_{max} = -n\ F\ V_r \tag{35}$$

where $n$ is the number of equivalents, or electrons per molecule of fuel, (in this case $H_2$ so $n = 2$), $F$ is the Faraday (96,493 Coulombs per equivalent) and $V_r$ is the thermodynamic reversible voltage of the reaction (1.229 for this reaction) [Liebhafsky and Cairns, 1968]. Thus, $W_{max}$ is:

$$W_{max} = (-2equiv)\ (96,493\frac{C}{equiv})\ (1.229V) = .24\frac{MJ}{mole} = 66\frac{W \cdot h}{mole} \tag{36}$$

Since we have seen tanks above that store 1 liter at 3000 psi (207 atm), we can see how much energy is available if the gas is hydrogen. Using the ideal gas law we find that there are:

$$n = \frac{p\ Vgr}{R\ T} = \frac{207 \times 10^5 \frac{N}{m^2}\ 10^{-3} \frac{m^3}{1liter}}{(8.31)(300)} = 8.3\frac{moles}{liter} \tag{37}$$

which means that we could theoretically get 548 $W \cdot h$ out of a liter tank of hydrogen, which would weigh about a kg (excluding the fuel cell assembly itself for now). Clearly this is a desirable energy source, especially since it is *generated* energy, not stored.

Unfortunately, this power is not attainable due to actual heat losses, unreacted fuel that diffuses out of the system, and mass transport problems, but efficiencies can be quite high.

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