CS 3240 Homework 3
Due: Thursday, March 9 by 5pm

Guidelines:

1. Work on the homework individually. Do not collaborate or copy from others

2. The homework is due on: Thursday, March 9 by 5pm. No late submissions will be entertained

3. Submit your homework in the box outside Prof. Pande's office at CCB 222 (College of Computing building)

4. Do not email your answers to either the Professor or the TA. Emailed answers will not be considered for evaluation

Question 1: Pumping Lemma and Regular Languages

You can use the pumping lemma and the closure of the class of regular languages under union, intersection and complement to answer the following question. Proofs should be rigorous. Note that for each of the questions below, you may or may not have to use the pumping lemma.

Note that the notation 0^m means "0 repeated *m* times". So the language of strings of the form 0^m such that $m \geq 0$ would contain strings like the null string 0, 00, 000, … (this is [0]*. Whereas the language of strings of the form 0^m such that $m \geq 1$ would be [0]+)

a. Is the language of strings of the form $0^m1^n0^m$ such that $m, n \geq 0$ regular? If it is regular, **prove** that it is regular. If it is not regular, **prove** that is not regular. Note that, a rigorous proof is needed. General reasoning or explanations that are not rigorous will not get full credit. (15 points)

b. Consider a language whose alphabet is from the set {a, b}. Is the language of *palindromes* over this alphabet regular? If it is regular, **prove** that it is regular. If it is not regular, **prove** that is not regular. Note that, a rigorous proof is needed. General reasoning or explanations that are not rigorous will not get full credit. (15 points)

Hint: A palindrome is a word such that when read backwards, is the same word. For example the word "mom" when read left to right is the same as it is when it is read right to left. In general, the first half, when reversed, yields the second half. If the length of the string is odd, the middle character is left as it is. For example, consider the word "redivider". Reversing "redi" yields "ider" and "v" is left as it is. For strings with alphabet {a, b}, "aaabaaa" is a palindrome but "abaaa" is not.

c. A language, whose alphabet is {a, b}, such that the strings of the language contain equal number of "ab" and "ba". Note that "aba" is part of the language, because the first letter and the second letter form "ab" and the second and third form "ba". Is this language regular? If it is regular, **prove** that it is regular. If it is not regular, **prove** that is not regular. Note that, a rigorous proof is needed. General reasoning or explanations that are not rigorous will not get full credit. (15 points)

d. The class of regular languages is closed under union. That is of A is a regular language and B is a regular language, then C is a regular language, where $C = A \cup B$. Note that $B \subseteq C$. (B is a subset of C). Let D be some subset of C (that is, $D \subseteq C$). In general, is D regular? If it is regular, **prove** that it is regular. If it is not regular, **prove** that is not regular. Note that, a rigorous proof is needed. General reasoning or explanations that are not rigorous will not get full credit. (15 points)

Question 2:

Consider the language described by the regular expression a+b*a, the set of all strings that has one or more a's followed by zero or more b's and ending in a single a.

a. Construct a NFA which recognizes this language. Note that you need to construct a *primitive NFA* using the constructions describe in class. (10 points)

b. Convert the above NFA to a DFA using $\in$ closure. Clearly indicate the steps of $\in$ closure. (20 points)

c. Convert the above DFA to an optimized DFA (10 points)