CS 3510 A **Fall 2005**

Mid-term Exam Announcements

First midterm exam will be held on **Wednesday, October 5**. The exam will be closed book. You may, however, bring one 8.5" x 11" sheet of paper as a crib sheet. Preparing a crib sheet can be a useful study aid, so take time in selecting material for it. You may use ONLY ONE SIDE of the paper and write as small as you like. No mechanical or electronic reproductions are allowed. You must hand in your crib sheet when you hand in your exam.

Material covered: You are responsible for the material covered in lecture and assigned for reading until Monday, October 3 (inclusive). In particular, it includes: asymptotic notation(O, Ω, Θ), methods for solving recurrences (substitution, divide-conquer recurrences and the master method), insertion sort, divide-conquer algorithms (merge sort, quicksort, Strassen's matrix multiplication, integer and complex number multiplication, median and order statistic: randomized and deterministic), lower bound for comparison sorting, graphs: representations, dfs and its applications, dag, topological sorting, bfs with applications, shortest paths algorithms (bfs, Dijkstra's alg and Bellman-Ford alg.), heaps.
Review: There will be a review in class on **Monday, Oct. 3**

The best way to study for this exam is to

- review the assignments, make sure you can solve the problems, practice solving similar ones;
- review your lecture notes.

Sample problems:

1. Prove that MergeSort algorithm has the same asymptotic running time ($= O(n \log n)$) when instead of dividing the array into two subarrays of the same size we divide it into three subarrays of equal size and sort them recursively. You can assume that $n = 3^k$.

2. You are given a set A of $n \geq 2$ distinct numbers in unsorted order. In $O(n)$ time, determine $x, y \in A$ such that
$$|x - y| \geq |w - z|,$$
for all $w, z \in A$.

3. What is the running time of the Quicksort algorithm when all elements in the input array have the same value. Justify your answer.

4. Let A be an array of n integers such that $A[i] \leq A[i+1]$ for all $1 \leq i \leq n-2$. Describe an algorithm for sorting the array A that uses $O(n)$ element comparisons.

5. TRUE or FALSE? Answer and justify briefly.

    - Let G be a connected, undirected, dense graph with no edge weights and let s and t be two vertices in G. Finding a shortest path from s to t requires $\Omega(n^2 \log n)$ time.

- Deciding if any input array of n numbers is sorted requires $\Omega(n^2)$ time.

6. Use DFS to give an algorithm for the following problem:

   - Given an adjacency list representation of an undirected graph $G = (V, E)$, design a linear $(O(|E|+|V|))$ time algorithm to decide if G is a tree. Why does your algorithm run in the claimed time bound?

7. Let $G = (V, E)$ be a directed graph representing a communication network. For every edge e we have a real value $r(e)$, where $0 \leq r(e) \leq 1$, which is the probability that the link e does not fail. Assume that the probabilities on the edges are independent. Describe an efficient algorithm to find a path in G that is most reliable (has the smallest probability of failure) between two given vertices.

8. Give an $n \log k$ time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all input lists. (use a MIN-HEAP for $k-$way merging.)

9. Problem 6-1/p. 142

10. Problem 6-2/p. 143

11. Give an efficient algorithm to count the total number of paths in a directed acyclic graph. Analyze your algorithm.

12. Show that to determine whether a directed graph G on n vertices, represented by the adjacency matrix, contains a **universal sink**- a vertex $v : d_{in}(v) = n - 1$ and $d_{out}(v) = 0$ takes time $O(n)$.