CS 3510: Design and Analysis of Algorithms
Fall 2008

Home work 2 // Due: Friday, September 19, 2008

1. (15 points) Exercise 2.22, page 75 of the text.

2. (15 points) Exercise 2.28, page 76-77 of the text.

3. *All arithmetic operations in this problem are done modulo* 41.

   (a) (10 points) Find a number ω such that ω satisfies the properties of a principal 4-th root of unity:

   - $\omega^4 = 1$.
   - $\omega^j \neq 1$ for $1 \leq j \leq 3$.
   - ω, ω^2, and ω^3 are distinct.

   i. Show that ω satisfies these properties.

   ii. Show that $1 + \omega + \omega^2 + \omega^3 = 0$.

   iii. Write down the matrix $M(w)$ whose (i,j)-th entry is: $(\omega)^{i \times j}$ for $0 \leq i,j \leq 3$.

   (b) (5 points) Find a number ω^{-1} such that $\omega \times \omega^{-1}$ is 1. Show that ω^{-1} satisfies the properties of a principal 4-th root of unity.

   (c) (5 points) Write down the matrix $M(\omega^{-1})$ whose (i,j)-th entry is: $(\omega^{-1})^{i \times j}$ for $0 \leq i,j \leq 3$. Show that $M(\omega^{-1}) \times M(\omega)$ is a diagonal matrix with all diagonal entries equal to 4.

   (d) (15 points) Using the matrices $M(\omega)$ and $M(\omega^{-1})$ show how to compute the product of the polynomials $3 - x$ and $2 + 3x$ using the following steps:

   - Evaluate the polynomial $3 - x$ at $(1, \omega, \omega^2, \omega^3)$ using the matrix $M(\omega)$. Show the values of the resulting vector.
   - Evaluate the polynomial $2 + 3x$ at $(1, \omega, \omega^2, \omega^3)$ using the matrix $M(\omega)$. Show the values of the resulting vector.
   - Form the point-wise product of the two vectors from the previous two steps. Show the values of the resulting vector.
   - Find the inverse-transform of the vector from the previous step using the matrix $M(\omega^{-1})$.