CS 6505: Computability & Algorithms

Homework #4, due in class Wednesday, February 17, 2010.

1. Let $CUT(G, k)$ be the language consisting of representations of a graph G and a natural number k, and G has a cut of size k. In other words, G contains a set of vertices S such that there are k edges $(u, v), u \in S, v \in \overline{S}$. Show that this language is in NP by giving a certificate that a particular (G, k) is in the language and a polynomial time method for using it to prove membership.

   Note: The certificate does not have to show that there does not exist a cut larger than k.

2. Let DISJOINT-PATHS$(G, (x_1, y_1), (x_2, y_2), \ldots, (x_k; y_k))$ be the language consisting of representations of a graph G and a list of pairs of nodes (x_i, y_i) such that G contains a path from x_i to y_i for all i and all such paths are vertex-disjoint. Prove the language is in NP.

3. A *context-sensitive grammar* is a set of production rules of the form

   $$\alpha A \beta \rightarrow \alpha \gamma \beta$$

   where $\alpha, \gamma, and \beta$ are strings, γ is non-empty, and A is a single (non-terminal) symbol, plus (possibly) one rule

   $$S \rightarrow \lambda$$

   where S does not appear on the right side of any production and λ is the empty string.

   The language produced by a given CSG is the set of strings which can be produced by beginning with the symbol S, and then repeatedly choosing a rule for which the current string matches the pattern on the left side and replacing it with the string given by the pattern on the right side. Let CSG(G, s) be the language consisting of representations of a context- sensitive grammar G and a string s such that G produces s. Prove that CSG(G, s) is PSPACE-complete by reducing to it from TQBF.

4. (bonus) Let PARALLEL-PATHS(G, s, t, k) be the language consisting of representations of a graph G, nodes $s, t \in V(G)$, and natural number k such that there exist k vertex-disjoint paths between s and t in G. Show that this language is in NP and also in co-NP.