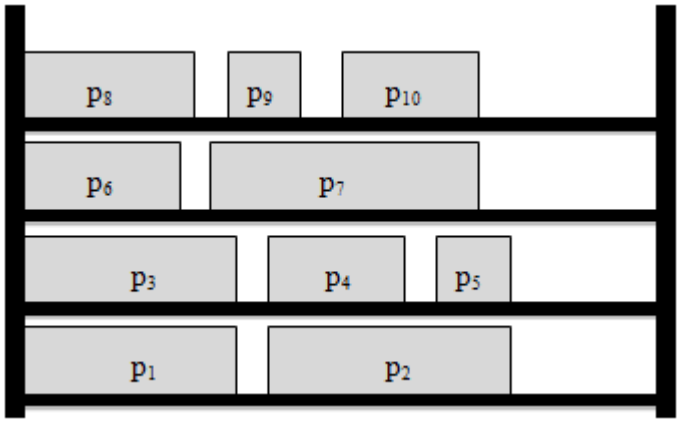
CS6505 Algorithms & Computability

Homework #5, Due in class on Fri. 2/26

1. Buy-N-Large has to sell quite a lot of small useless goods, and to maximize throughput in their warehouses, they use giant rotating shelves. As item p_i arrives, they place it on the top shelf, exactly 1 inch to the right of item p_{i-1}, or, if the shelf is empty, aligned all the way to left side of the shelf. Each item p_i is exactly w_i inches wide and each shelf is W inches wide. When they are done putting items on a shelf (which may happen at any time), they rotate the shelf down and begin stocking the shelf above. No item may hang off the edges or the shelves will jam when they reach the un-stocking robots at the bottom of the shelf device (see diagram).
   At some point in time, the top shelf is empty. Call the next item to arrive p_1. In order to minimize the motion by the robots that stock and unstock the shelves, we want to stock items $p_1 \dots p_P$ in such a way that the sum of squares of the distance from the rightmost element of a shelf to the right edge of the shelf is minimized.
   Give an algorithm that decides which items go on which shelves (which is equivalent to deciding between which items to start stocking the next shelf) and minimizes this value. Demonstrate that it is correct.



2. Given a graph G with nonnegative edge lengths $l(u, v)$, and a pair of nodes s and t, find a path between s and t that minimizes the maximum length edge on the path. Give an algorithm and prove its correctness.

3. The amount of energy generated by a hydroelectric dam decreases over time as silt builds up behind it, but the dam can be closed and the silt flushed and dredged out until the dam is restored to its peak efficiency. Assume that in the ith year since a dam was last flushed, it will output a guaranteed w_i kWh of energy, where w_i is a strictly decreasing sequence. In addition, it takes a whole year to fully flush the silt from a dam. Assume that you can accurately predict the energy demand for the next n years

to be $d_1, \ldots, d_n$. Give an optimal policy for operating the dam so as to maximize the amount of demand satisfiied.