CS6505 Homework 6. Due Fri, Mar 5th

1. Given a function $f : [0,1]^n \rightarrow \mathbb{R}$ which can be computed in polynomial time, show that the problem of determining whether $\exists x_1 \ldots x_n$ such that $f(x_1, \ldots, x_n) > 0$ is NP-complete.

2. You are given two multisets X and Y of integers. You can move any subset of numbers from X to Y and any subset of numbers from Y to X. Show that determining whether there is such a re-arrangement for which there exists an integer α such that

$$\sum_{x \in X} \sum_{y \in Y} (x, y) = \alpha(|Y|, |X|)$$

   is NP-complete.

3. Given a graph $G = (V, E)$, and a number k, the bounded-degree spanning tree problem is to find a spanning tree with the property that the degree of every vertex in the tree is at most k. Show that the decision version of bounded-degree spanning tree is NP-complete.

4. For a graph $G(V, E)$, and a subset of vertices S, the density of the subgraph induced by S is the ratio of the number of edges with both endpoints in S to the number of pairs of vertices in S. Given a graph $G = (V, E)$, an integer k and a number δ between 0 and 1, show that the problem of determining whether the graph has a subgraph with at least k vertices and density at least δ is NP-complete.