CS 6505, Complexity and Algorithms

Homework 8, due Friday, March 19th in class.

1. Suppose that we have equations with 3 variables in the form

$$x_i + x_j + x_k = b \pmod 2,$$

where $b \in \{0, 1\}$, x_i, x_j, x_k are binary variables, and the equations are modulo 2.

   (a) Suppose that we are given a system of m such equations, and each equation has exactly three distinct variables selected from the set $X = x_1, \ldots, x_n$. Give a simple randomized algorithm that satisfies at least half of the equations.

   (b) Show how to make the algorithm from part (a) deterministic.

   (c) Now suppose that we are given k variables per equation. What fraction of the equations can you satisfy?

2. Suppose that we want to check if two graphs on the same set of vertices are equal (i.e., they have the same set of edges). The first graph, G, is given to us explicitly. The second graph, H, is given via a blackbox that answers the following question: Given a subset of vertices S, what is the number of edges of H that have both vertices in S?

   Give a simple randomized algorithm that determines whether $G = H$ and bound its probability of error, number of queries to the blackbox and total computation time.

3. Suppose we are given two n-bit binary strings, $x, y \in \{0, 1\}^n$. Define the relative distance $\Delta(x, y)$ between two strings as follows:

$$\Delta(x, y) = \frac{|\{j : x_j \neq y_j, 1 \leq j \leq n\}|}{n}$$

   Now suppose that x and y are stored on computers that are very far apart, and we would like to determine if they are the same. Transferring a single string to the other computer may take too long. Give a randomized algorithm to approximate $\Delta(x, y)$ to within a factor of 2, i.e., the algorithm outputs an answer d such that $\Delta(x, y) \leq d \leq 2\Delta(x, y)$. Give the number of bits that must be transmitted in your approach as well as the probability of error of the algorithm. [Bonus: show how to do this for arbitrarily small approximation, i.e., the algorithm outputs d such that $\Delta(x, y) \leq d \leq (1 + \epsilon)\Delta(x, y)$.]