CS 6505: Computability & Algorithms

Homework 1, due in class Wednesday, January 27, 2010.

1. Prove that $\{(x, y, z) | x, y, z \in \mathbb{N}\}$ is countable.

2. a. Show that it is undecidable whether a Turing Machine halts on the empty string (or null input).

   b. Show that it is undecidable whether a Turing Machine accepts a finite language.

3. Given a currency and an amount in that currency, one problem is to find the minimum cardinality set of currency items (notes or coins) of the given amount. For example, in US currency, we would use 1c, 5c, 10c, 25c, $1, $5, $10, $20, and $100.

   a. Show that the greedy algorithm which picks the largest currency item that is smaller than the remaining amount works for US currency.

   b. Make up a currency for which the greedy algorithm does not work.

   c. Give a property of currency which would guarantee that the greedy algorithm works.