Strassen's algorithm

Note Title 3/3/2010

$$(AB)_{ij} = \sum_k A_{ik} B_{kj}$$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix}$$

Here $A_{11}, A_{12}, \ldots$ are matrices

$$f(n) = 8f\left(\frac{n}{2}\right) + c \cdot n^2$$

$$= O\left(n^{\log_3 8}\right) = O(n^3).$$

Can we do this faster?

Let's try the 2×2 case

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

8 multiplications 4 additions

Which one should we try to save

Suppose we can do this in m mult. l additions

$$f(n) = m f\left(\frac{n}{2}\right) + l \cdot \left(\frac{n}{2}\right)^2$$

$$= O\left(l \cdot n^{\log_2 m}\right) \quad \text{assuming } m \geq 4$$

using Master's thm

Alternatively # mult $= 1 \cdot n \times n$

$$= m \left(\frac{n}{2} \times \frac{n}{2}\right) = m^2 \left(\frac{n}{4} \times \frac{n}{4}\right) \cdots = m^k \text{ mult.} \qquad n = 2^k$$

Additions $= l\left(\frac{n}{2}\right)^2 + l \cdot m \left(\frac{n}{4}\right)^2 + l \cdot m^2 \left(\frac{n}{8}\right)^2 \longrightarrow \leq l\, m^k.$

So # multiplications is much more important.

STRASSEN

$$M_1 = (a_{11} + a_{22})(b_{11} + b_{22})$$

$$M_2 = (a_{21} + a_{22})\, b_{11}$$

$$M_3 = a_{11}(b_{12} - b_{22})$$

$$M_4 = a_{22}(b_{21} - b_{11})$$

$$M_5 = (a_{11} + a_{12})\, b_{22}$$

$$M_6 = (a_{21} - a_{11})(b_{11} + b_{12})$$

$$M_7 = (a_{12} - a_{22})(b_{21} + b_{22})$$

7 mult!

$$C_{11} = M_1 + M_4 - M_5 + M_7$$

$$C_{12} = M_3 + M_5$$

$$C_{21} = M_2 + M_4$$

$$C_{22} = M_1 - M_2 + M_3 + M_6$$