Problem 1

Of the entire population, 2% has a certain disease X. A test Y, which indicates whether or not a person has the disease, is not 100% accurate. If a person has the disease, there is a 6% chance that it will go undetected by the test. However, there is also a 9% chance of "false alarm" (meaning that the person does not have the disease but the test indicates otherwise). A person Z takes a test which later comes out positive (meaning that the test says he has the disease). What is the probability of this person having the disease in reality?

Let $D$ be "having the disease"
+ be "test positive"

We are given the following information:
$P(D) = 0.02$
which implies $P(\text{not } D) = 0.98$
$P(\text{not } + \mid D) = 0.06$
which implies $P(+ \mid D) = 0.94$
$P(+ \mid \text{not } D) = 0.09$

First, we compute $P(+)$
$= P(+ \text{ AND } D) + P(+ \text{ AND } \text{not } D))$
$= P(+ \mid D) P(D) + P(+ \mid \text{not } D) P(\text{not } D)$
$= 0.94 \times 0.02 + 0.09 \times 0.98$
$= 0.107$

We would like to know $P(D \mid +)$
$= P(+ \mid D) \times P(D) / P(+)$
$= 0.94 \times 0.02 / 0.107$
$\approx 0.1757$
Problem 2

Consider the following Bayesian network:

a) Are D and E necessarily independent given evidence about both A and B?

No. The path D-C-E is not blocked.

b) Are A and C necessarily independent given evidence about D?

No. They are directly dependent. The path A-C is not blocked.

c) Are A and H necessarily independent given evidence about C?

Yes. All paths from A to H are blocked.
Problem 3

Consider the following Bayesian network. A, B, C, and D each could have a value of either true or false. If we know that A is true, what is the probability of D being true?

\[ P(D | A) = \sum_{(b,c) \in B \times C} P(D | (B,C) = (b,c)) \times P((B,C) = (b,c) | A) \]

\[ = P(D | B \text{ and } C) \times P(B \text{ and } C | A) + \]
\[ P(D | B \text{ and } \lnot C) \times P(B \text{ and } \lnot C | A) + \]
\[ P(D | \lnot B \text{ and } C) \times P(\lnot B \text{ and } C | A) + \]
\[ P(D | \lnot B \text{ and } \lnot C) \times P(\lnot B \text{ and } \lnot C | A) \]

\[ = (0.3 \times 0.2 \times 0.7) + (0.25 \times 0.2 \times 0.3) + (0.1 \times 0.8 \times 0.7) + (0.35 \times 0.8 \times 0.3) \]

\[ = 0.042 + 0.015 + 0.056 + 0.084 \]

\[ = 0.197 \]