Artificial Intelligence
Value of Information

Russell and Norvig
Chapter 16.6 - 16.7

“Decision Trees”

so far:

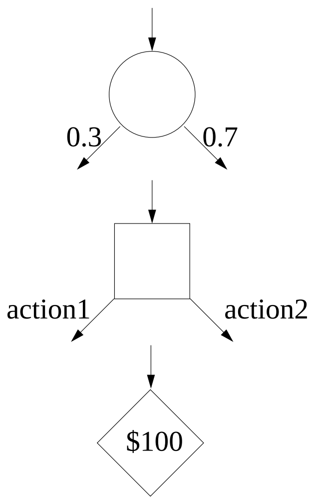
Bayes Nets
probabilistic reasoning

now:

“Decision Trees”
probabilistic reasoning
including costs and actions

- similar to Influence Diagrams (Decision Networks)
- similar to Game Trees
- not similar to Decision Trees used in machine learning

“Decision Trees”



Chance Node
= average

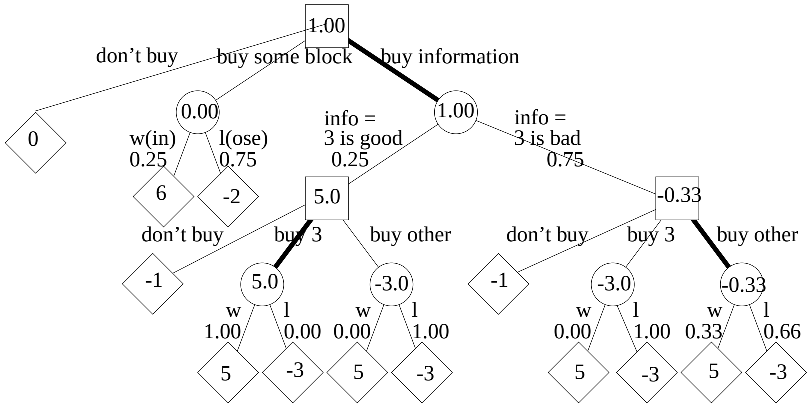
Decision Node
= take max (or min)

Utility Node

Value of Information

- should a doctor perform a certain lab test
- should we by the “Wall Street Journal”
  to make better business decisions
- should a robot spend time localizing itself
  to improve its navigation performance

An oil company is hoping to buy one of 4 indistinguishable blocks of ocean drilling rights, exactly one of which contains oil worth 8 dollars. The price of each block is 2 dollar. A seismologist offers the company the results of a survey of block number 3, which indicates definitevely whether the block contains oil, for 1 dollar. Should the company buy the information? How much would it pay at most for the information?



Value of Information

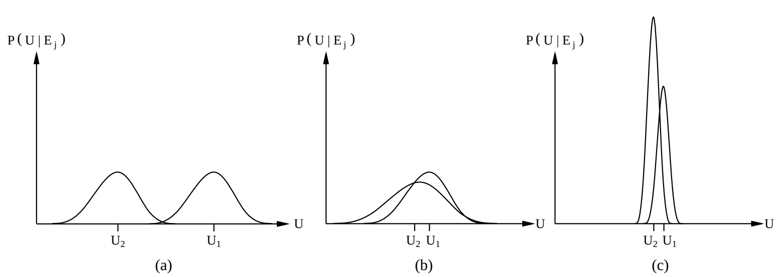
= how much you should pay at most for the information
= performance improvement achievable with the information

get the information
(for example, perform the test)

iff

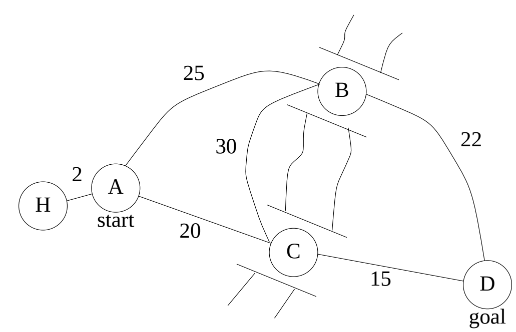
value of information >= cost of information

Value of Information



notice:
information has value only
to the extent that it is likely to cause a change of plan
and to the extent that the new plan will be significantly better than the old one

Example



In similar situations in the past, the robot experienced that 4 out of 5 times bridge C was out and only 1 out of 5 times bridge B was out. The robot has a short-range sensor that tells it with 100 percent reliability whether a bridge is out. This sensor can only be used when the robot is directly in front of the bridge. The long-range sensor of the robot is unreliable. It errs with a probability of 10 percent, that is, suggests that the broken bridge is operable and the other bridge is broken.

Artificial Intelligence CS6361

Utility Theory for Risk-Sensitive Decision Making

Russell and Norvig

Chapter 161 - 16.3

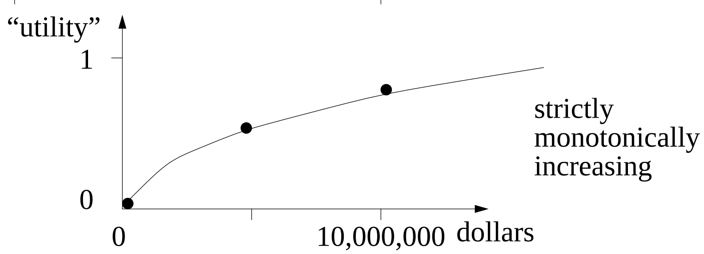
Utility Theory (1)

probability 1.0: win 4,500,000 dollars

probability 0.5: win 10,000,000 dollars
probability 0.5: win 0 dollars

Utility Theory (2)

prob	reward	exp reward	utility	exp utility
1.0	4,500,000	4,500,000	0.6	0.6
0.5	10,000,000	5,000,000	0.8	0.4
0.5	0		0.0	



utility function: money -> utility
its existence follows from simple axioms

Utility Theory (3)

utility function 1:
$u(x)$

lottery 1:
prize x_i with probability p_i

utility function 2:
$u'(x) = m\, u(x) + n$ with $m > 0$
(positively linear transformation)

lottery 2:
prize y_j with probability q_j

utility function 1:
compare $\Sigma_i\, p_i\, u(x_i)$ and $\Sigma_j\, q_j\, u(y_j)$

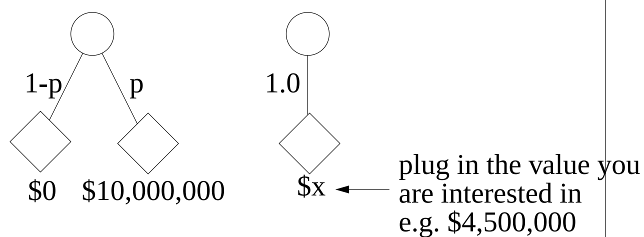
utility function 2:
compare $\Sigma_i\, p_i\, u'(x_i)$ and $\Sigma_j\, q_j\, u'(y_j)$
compare $\Sigma_i\, p_i\, (m\, u(x_i) + n)$ and $\Sigma_j\, q_j\, (m\, u(y_j) + n)$
compare $m\, (\Sigma_i\, p_i\, u(x_i)) + n$ and $m\, (\Sigma_j\, q_j\, u(y_j)) + n$
compare $\Sigma_i\, p_i\, u(x_i)$ and $\Sigma_j\, q_j\, u(y_j)$

utility functions are defined only up to positively linear transformations

Utility Theory (4)

people have different utility functions
how to elicit them from decision makers?

what is the value of p that makes you indifferent between:



$$(1-p)\ u(\$0) + p\ u(\$10{,}000{,}000) = u(\$x)$$
$$(1-p)\ 0 + p\ 1 = u(\$x)$$
$$p = u(x)$$

Utility Theory (5)

normative theory, not descriptive theory

A: probability 0.80: win $4,000
B: probability 1.00: win $3,000

C: probability 0.20: win $4,000
D: probability 0.25: win $3,000

Choosing B over A, and C over D is inconsistent.
Set u($0) = 0 and solve!