derive a new estimate for the right-hand sides. This is the basic premise of Ikeuchi's method for Shape from Shading and is shown below.

Algorithm 1: Shape from Shading

Step 0. *k=0.* Pick an initial estimate of p and q near boundaries $p^o(x, y)$ and $q^o(x, y)$.

Step 1. *k = k+1;* compute

$$p^k = p_{av}^{k-1} + T\frac{\partial R}{\partial p}$$

$$q^k = q_{av}^{k-1} + T\frac{\partial R}{\partial q}$$

Step 2. If the sum of all E's is sufficiently small, stop, else continue

A loose end in this algorithm is that boundary conditions must be specified. These are values of p and q determined a prior that remain constant throughout each iteration. The simplest place to specify a surface gradient is at an occluding contour, where the gradient is nearly 90 degrees to the line of sight. Unfortunately p and q are infinitie at these points. However, if we use a different coordiate system for gradient space (i.e., using a Gaussian sphere, yielding spherical coorddinate system), we can avoid such boundary conditions.

In this system the radiance may be described in terms of spherical coordinates (θ, ϕ). For a lambertian surface

$$R(\theta, \phi) = \cos\theta\cos\theta_s + \sin\theta\sin\theta_s\cos(\phi - \phi_s) \qquad \mathbf{5}$$

At an occluding contour $\varphi = \frac{\pi}{2}$ and $\theta = \operatorname{atan}\frac{\partial y}{\partial x}$.

To use the (θ, ϕ) formulation instead of *(p,q)* is as easy as a simple substitution.

2.0 References

[Ikeuchi 1980] Ikeuchi, K., "Numerical shape from shading and occluding contours in a single view", AI Memo 566, AI Lab, MIT, revised February 1980.

[Strang 1986] Strang, G., *Introduction to Applied Mathematics*, Wellesley-Cambridge Press, 1986.

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Notes on Shading

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1.0 Shape from Shading by Relaxation

Combining local information allowed improved estimation for edges and for stereo disparity. In a similar manner local information can help with computing surface orientation [Ikeuchi 1980]. Basically, the reflectance equation provides one constraint on the surface orientation and another is provided by the heuristic requirement that the surface be smooth

Suppose there is an estimate of surface normal at point $(p(x, y), q(x, y))$. If the normal is not accurate, the the relectivity equation $I(x, y) = R(p, q)$ will not hold. Thus it seems reasonable to seek *p* and *q* that minimize $(I - R)^2$. The other requirement is that *p(x,y)* and *q(x,y)* be smooth. A condition that can be measured by the partial derivates squared, *i.e.*, $p_x^2, p_y^2, q_x^2, q_y^2$. Smoothness constraint requires both of the above terms to be small. This can be done by reducing the error at a point *x,y*.

$$E(x, y) = (I(x, y) - R(p, q))^2 + \lambda(p_x^2 + p_y^2 + q_x^2 + q_y^2). \qquad \textbf{1}$$

Where the λ is the Lagrange multiplier [Strang 1986] and is used to incorporate the smoothness constraint.

Differentiating Equation (1) above with respect to p and q and approximating the derivatives numerically gives the following:

$$p(x, y) = p_{av}(x, y) + T(x, y, p, q)\frac{\partial R}{\partial p},$$
$$q(x, y) = q_{av}(x, y) + T(x, y, p, q)\frac{\partial R}{\partial q} \qquad \textbf{2}$$

where,

$$T(x, y, p, q) = \left(\frac{1}{\lambda}\right)[I(x, y) - R(p, q)] \qquad \textbf{3}$$

and.

$$p_{av}(x, y) = \frac{1}{4}[p(x+1, y) + p(x-1, y) + p(x, y+1) + p(x, y-1)] \qquad \textbf{4}$$

and a similar equation for q_{av}. Equations (4) lend thmesleves to a solution using Gauss-Siedal method: calculate the left hand sides with an estimate for p and q and use them to