## 11.8 Alternative Kalman equations

Having shown that the covariance matrix can be updated via the previous equation it is possible to formulate an alternative Kalman gain, as follows;

$$K_k = P'_k H^T \left(H P'_k H^T + R\right)^{-1} \tag{11.53}$$

Inserting $P_k \times P_k^{-1}$ and $R \times R^{-1}$;

$$\begin{aligned} K_k &= P_k P_k^{-1} P'_k H^T R^{-1} R \left(H P'_k H^T + R\right)^{-1} \\ &= P_k P_k^{-1} P'_k H^T R^{-1} \left(H P'_k H^T R^{-1} + I\right)^{-1} \\ &= P_k \left(I + H^T R^{-1} H P'_k\right) H^T R^{-1} \left(H P'_k H^T R^{-1} + I\right)^{-1} \\ &= P_k H^T R^{-1} \left(I + H^T R^{-1} H P'_k\right) \left(I + H P'_k H^T R^{-1}\right)^{-1} \\ &= P_k H^T R^{-1} \end{aligned} \tag{11.54}$$

Replacing $P_k$ with the inverse of equation 11.50;

$$K_k = \left[H R^{-1} H^T + P_k'^{-1}\right]^{-1} H^T R^{-1} \tag{11.55}$$

Which is the same as the gain calculated from the chi-square equations, confirming that the gains are indeed equivalent.

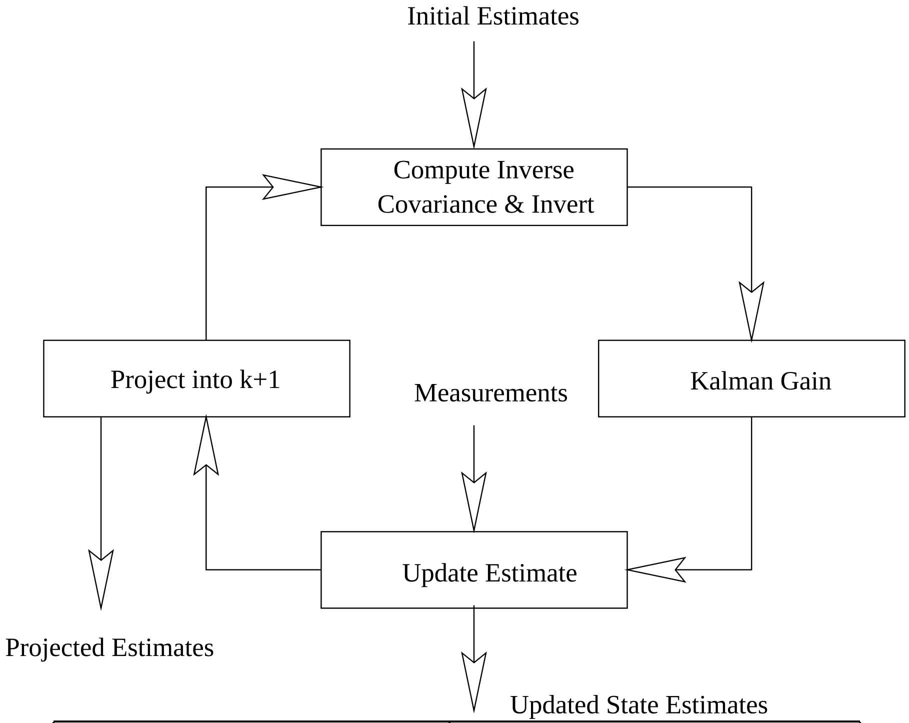
Although an alternative recursive algorithm has been developed the objective was to demonstrate the relationship between the Kalman filter and the chi-square statistic, showing how the Kalman filter embodies this statistic. The diagram of figure **??** shows how the alternative set of filter equations may be used to implement a Kalman filter. This form of the filter may be attractive due to the simplified gain calculation and some authors have been able to use this form of the filter in a distributed implementation [**?**]. However in this form the filter requires two matrix inversions which can be a computational burden, particularly when large matrices are involved. Thus the preferred implementation here is that given in figure 11.5.

## 11.9 Conclusions

This tutorial has shown how the Kalman filter may be derived from the desire to minimise the mean squared error of a signal prediction. Several points in the derivation have been emphasised;

- The minimisation of the mean squared error is shown to be applicable when the expected errors on the signal are distribution as a Gaussian. Under such conditions the minimisation of the mean squared error between the data and the data prediction leads to the development of a *maximum likelihood statistic*
- It has been shown how the Kalman filter can be thought of in terms of a chi-squared minimiser by deriving an alternative form of the Kalman filter which highlights its statistical constructs including the processes of error propagation and data combination. This derivation leads to a common, alternative set of filter equations.

In summary, although the Kalman filter is optimal in the mean-squared error sense, it is limited, practically by the quality and accuracy of the model which is embedded within it. However, without an appropriate model the filter is unable to perform the task for which it is designed. The following sections describe a new statistical approach to the solution of this problem.




| Description | Equation |
|---|---|
| Compute Inverse Covariance | $P_k^{-1} = HR^{-1}H^T + P_k'^{-1}$ |
| Kalman Gain | $K_k = P_k H^T R^{-1}$ |
| Update Estimate | $\hat{x}_k = \hat{x}_k' + K_k(z_k - H\hat{x}_k')$ |
| Project into $k+1$ | $\hat{x}_{k+1}' = \Phi \hat{x}_k$ <br> $P_{k+1} = \Phi P_k \Phi^T + Q$ |

Figure 11.2: Alternative Kalman filter recursive algorithm

# Bibliography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