ARC Proposal: Models for Routing and Social Complex Networks

Georgios Amanatidis, 4th Year PhD Candidate in Math/ACO amana@math.gatech.edu
Advisor: Milena Mihail, Associate Professor, CoC mihail@cc.gatech.edu

Models for complex communication and social networks are the object of intense study due to their simulation and predictive power. The first generation of such network models consisted of random graphs with skewed degree distributions. However, in the last few years, objections have been raised concerning the generality of such models.

A. The Networking Conext: In networks designed under optimization, including many technological networks, fundamental metrics disagree with random graph models. Most notably, [ADGW03, LAWD04] (see also [New02, New03]), argued that several Internet topologies, which are highly optimized for cost and performance, are sharply different than power law random graphs. In particular, random graphs construct a dense core of nodes with very high degrees, while nodes of smaller degree are in periphery of the network. On the other hand, highly optimized topologies place low degree/high bandwidth routers at the center of the network, while high degree nodes are placed in the periphery to split the signal manyways towards the end users. Going one step further, [MKF+06, MKFV06] argued that a determining metric for a graph of given degrees to resemble a real network topology, is the specific number of of links between vertices of different degree classes.

The **joint degree matrix realizability** problem formalizes the approach of [MKF+06, MKFV06], and generalizes the classical theory of Erdős-Gallai and Havel-Hakimi on realizability and connected realizability of degree sequences. Let $V=[n]$ be a set of vertices. Let $\mathbb{V}=\{V_1,V_2,\ldots,V_k\}$ be a partition of V denoting subsets of vertices with the same degree. Let d be function $d:\cup_{i=1}^{k}V_i \rightarrow \mathbb{N}$ denoting the degrees of vertices in class V_i. Let $D=(d_{ij})$ be a $k \times k$ matrix denoting the number of edges between V_i and V_j. The problem is, given $\langle \mathbb{V}, d, D\rangle$, decide if there is a simple graph G on V that satisfies d and D. For networking purposes, it is also important to construct a connected such graph. Together with Bradley Green (Math/ACO student) and Milena Mihail we have obtained necessary and sufficient conditions for realizability and connected realizability of an instance $\langle \mathbb{V}, d, D\rangle$, and polynomial time algorithms to construct realizations without or with the connectivity restriction, or certificates that such realizations do not exist (in preparation for submission to FOCS 08). We note that the connected realization problem is highly technical and vastly different from its corresponding Erdős-Gallai counterpart.

We wish to work on the following natural generalizations; all of them are mathematically interesting and meaningful in the networking context: **(1)** Let $\mathbb{E} \subseteq V \times V$. Is there a realization which does not contain any edge in $\mathbb{E}$ or contains all edges in $\mathbb{E}$? **(2)** More generally, let c be a cost function on all potential edges $c: V \times V \longrightarrow \mathbb{N}_0$. Find a mincost such realization. **(3)** Find a uniformly random realization of $\langle \mathbb{V}, d, D\rangle$. There is natural Markov chain on the set of all realizations, however some non-trivial augmentation of the state space will be required to establish rapid mixing. **(4)** Define a joint degree matrix problem, where conditions on the number of edges do not involve only classes V_i of vertices of the same degree, but arbitrary partitions of vertices. For problems **(1)** through **(4)** above, we are exploring extensions of our algorithms, LP-based approaches and reductions to some better understood combinatorial problem(s) (such as matching or flow). **(5)** In general, the network topology generation problem consists of a set of network instances as the network has grown in the past, which must be followed by a prediction and construction of the network in the future. Therefore, for networking purposes, it is important to predict joint degree distributions from observations of smaller instances. We wish to also work in this direction.

B. The Social Network Context: Many networks, especially social networks, consist of selfish agents, and are thus inherently dynamic. An agent may change some of its links, if such changes improve the utility that this agent derives from the network. Economists and computer scientists have introduced several natural incentive based models [DW05, Jac08]. However, most of the work has focused on comparing the structure and cost of the incentive based formed network to a globally optimum such network (aka price of anarchy [NRTV07]: Part III: "Quantifying the Efficiency of Equilibria"). On the other hand, it is natural to ask (see also [Kle06]): **Are there incentive based network formation processes that stabilize in networks whose phenotypes (degree distribution, connectivity, etc) resemble real networks?** Such questions remains vastly unanswered.

Together with Bradley Green and Milena Mihail, we have isolated two popular network formation processes, namely the "connections model" and the "coauthor model" of Jackson and Wolinsky [DW05, Jac08], and are studying modifications that result in stable solutions with richer graph theoretic characteristics that those of the original models (such as variety of degrees and expansion). We wish to pursue this line of work and obtain well incentive based network formation processes which stabilize in graphs resembling real networks.

References

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