CS 1155: Understanding and Constructing Proofs

Spring 1999

Home work 6// Due: Friday, May 21, 1999

1. (10 points) Exercises 4.2, problem 18, page 211 of the text.

2. (10 points) Exercises 4.2, problem 22, page 211 of the text.

3. (15 points) Exercises 4.4, problem 8, page 233 of the text.

4. (15 points) Exercises 4.4, problem 12, page 234 of the text.

5. (10 points) Consider the definition of a *Fibonacci sequence* $FIB(n)$ in example 3 (a), page 228 of the text. Prove, by induction, that for all integers $k \geq 1$,

$$FIB_{k+2} = 1 + \sum_{i=1}^{k} FIB_i.$$

6. (10 points) Let $m = \alpha + \beta$ and $a = \alpha\beta$, where $m \neq 1$ and $\alpha \neq \beta$. Let $A_2 = m - \frac{a}{m-1}$. For $k > 1$, let $A_{k+1} = m - \frac{a}{A_k}$. Prove that, for $n \geq 2$,

$$A_n = \frac{(\alpha^{n+1} - \beta^{n+1}) - (\alpha^n - \beta^n)}{(\alpha^n - \beta^n) - (\alpha^{n-1} - \beta^{n-1})}.$$

7. (15 points) Let R be a binary relation on a set S with n elements. Let $R_0 = R$. Define R_{i+1} from R_i as follows:

$$R_{i+1} = R_i \cup \{(a,c) \mid (a,b) \in R_i \text{ and } (b,c) \in R_i\}.$$

   Show that if $R_i = R_{i+1}$ for some $i \geq 0$, then $R_i = R_j$ for all $j > i$.

8. Consider the recursive definition of the language L as follows: L consists of all strings over $\{0,1\}$ obtained from the basis step by a finite number of applications of the recursive step:

   Basis: The empty string is in L;

   Recursive: If $x \in L$ then $11x0 \in L$.

   (a) (5 points) How many strings of length k exist for each $k \geq 0$?

   (b) (10 points) Prove, by induction, that for every string $w \in L$ the number of 1's is twice the number of 0's.