CS 1155: Understanding and Constructing Proofs

Spring 1999

Home work 7// Due: Friday, May 28, 1999

1. Consider the following recurrence equation: $s_n = 6s_{n-1} - 11s_{n-2} + 6s_{n-3}$, with $s_0 = 3$, $s_1 = 6$, and $s_2 = 14$.

   (a) (5 points) What is the characteristic equation of this recurrence?

   (b) (5 points) What are the roots of the characteristic equation?

   (c) (10 points) What is the solution of the recurrence? Prove your answer.

2. (10 points) Consider the sequence defined in example 4, page 239 of the text. The characteristic equation has only one root, namely $r = 3$. (Therefore, for the solution of this recurrence, part (b) of Theorem 1 is applicable.) The proof of part (a) of Theorem 1 has two steps: (a) a basis step where the constants C_1, C_2 are determined, and (b) an inductive step. Which of these two steps in the proof of part (a) of Theorem 1 fails for this example? Why?

3. (15 points) Consider the matrices $M^1, M^2, and M^n$ defined on page 231 of the text book. Using these definitions, prove that, for all $n \geq 1$,

$$FIB(n+1).FIB(n-1) - FIB^2(n) = (-1)^n.$$

4. (10 points) Exercises 4.5, problem 10, page 244 of the text.

5. (10 points) Exercises 4.5, problem 12 (c), and (d), page 245 of the text.

6. (10 points) Exercises 4.5, problem 19 (b), page 245 of the text.

7. (10 points) Exercises 4.6, problem 12(d), page 251 of the text.

8. Let p, q be two propositions. Consider the following recursive definition of the set E of well-formed conjunctive and disjunctive Boolean expressions using p and q:

   Basis: $p, q \in E$.

   Recursive Step: If e_1, e_2 are expressions in E, then $(e_1 \vee e_2) \in E$ and $(e_1 \wedge e_2) \in E$.

   An expression is in E iff it is obtained from the basis by a finite number of applications of the recursive step.

   (a) (5 points) What are the expressions obtained after two applications of the recursive step to the set of expressions defined by the basis?

   (b) (10 points) For an expression e, let $P(e)$ denote the number of occurrences of the propositions p and q in e and let $O(e)$ denote the number of occurrences of the operators $\vee$ and $\wedge$ in e. Prove, by induction, that for every expression $e \in E$, $P(e) = 1 + O(e)$.