

CS 1155: Understanding and Constructing Proofs

Spring 1999

Home work 8// Due: Friday, June 4, 1999

1. Consider the following recursive definition of a tree:

Basis step Let r be a vertex. Then, the vertex set $V = \{r\}$, and the edge set $E = \phi$ is a tree with root r and leaf set $L = \{r\}$.

Recursive step Let T' be a tree with vertex set V' , edge set E' and leaf set $L' = \{v_1, v_2, \dots, v_n\}$.

Let $\{u_1, u_2, \dots, u_{2n}\}$ be a set of vertices not in V' . Then, the vertex set $V = V' \cup \{u_1, u_2, \dots, u_{2n}\}$ and the edge set $E = E' \cup \{\{v_1, u_1\}, \{v_1, u_2\}, \dots, \{v_i, u_{2i-1}\}, \{v_i, u_{2i}\}, \dots, \{v_n, u_{2n-1}\}, \{v_n, u_{2n}\}\}$ is a tree with leaf set $L = \{u_1, u_2, \dots, u_{2n}\}$.

- (a) (5 points) Draw the graph obtained by applying the recursive step three times starting from the basis.
 - (b) (10 points) Prove, by induction, that the number of vertices in the tree that results by applying the recursive step k times starting from the basis is $2^{k+1} - 1$.
2. (15 points) Let $f(x_1, x_2, \dots, x_n)$ be the Boolean function on B^n that takes the value 1 at $(a_1, a_2, \dots, a_n) \in B^n$ iff an even number of the entries a_1, a_2, \dots, a_n are 1. Show that any any Boolean expression in the sum-of products of literals form for f is a sum of at least 2^{n-1} products and each of these products must have n literals.
 3. (10 points) Exercises 9.1, problem 9, (b), (d), page 510 of the text.
 4. (10 points) Exercises 9.3, problem 4 (a), (b), page 524 of the text.
 5. (5 points) Exercises 9.3, problem 8 (b), page 525 of the text.
 6. (10 points) Exercises 9.3, problem 8 (b), page 525 of the text.
 7. (10 points) Exercises 9.3, problem 11 (b), page 525 of the text.
 8. (10 points) Exercises 10.3, problem 1 (b), page 576 of the text.
 9. (10 points) Consider the matrix A defined for the relation R in problem 5 (a), page 577 of the text. Prove, by induction, that the matrix for R^n is $A * A$.
 10. (5 points) Exercises 10.3, problem 5 (b), page 577 of the text.