CS 1155: Understanding and Constructing Proofs

Spring 1999

Quiz 3 // May 21, 1999

(Solution sketch)

1. (10 points) Let Σ be a finite alphabet. Say that a string x is related to a string y if length of x is equal to the length of y. This is an equivalence relation on Σ^*.

   (a) Does it have a finite number of equivalence classes? Why?

   No. Σ^* has 2^n strings of length n for each integer $n \geq 0$. Therefore, there is one equivalence class for each natural number n.

   (b) Describe the equivalence classes defined by this relation?

   The equivalence classes are Σ^k for each natural number k. (Here Σ^k is the set of all strings of length k.)

2. (10 points) Let A be a set. The partition of A is defined to be a collection of pair-wise disjoint subsets of A whose union is A.

   (a) Answer True/False with a very brief justification: The power-set of A is a partition of A.

   Although the union of the sets in the power-set of a set A is A, the power-set is not a disjoint collection of subsets.

   (b) Let R be an equivalence relation on $\mathcal{Z}$, the set of integers defined as follows: An integer m is related to an integer n if they have the same remainder when divided by 3. Describe the partition of Z defined by this equivalence relation.

   The partition of Z is defined by the three equivalence classes: the set of integers that leave a remainder 0 when divided by 3, the set of integers that leave a remainder 1 when divided by 3, and the set of integers that leave a remainder 2 when divided by 3,

3. (10 points) Let $\mathcal{Z}(4)$ denote the set $\{0, 1, 2, 3\}$. Define the operation $*_4$ on $\mathcal{Z}(4)$ as follows: for $a, b \in \mathcal{Z}(4)$, $a *_4 b = (a \times b)\ MOD\ 4$. Using this definition, write down the multiplication table for $\mathcal{Z}(4)$.

	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

4. (10 points) What is wrong with the following induction proof:

   Claim: Let $s_n = 2s_{n-1}$ and $s_0 = 1$. Then, $s_n = 2^{n+2}$.

   Proof: Suppose the claim is true for $n-1$. That is, $s_{n-1} = 2^{n+1}$. Then, the claim is true for n, since $s_n = 2s_{n-1} = 2 \times 2^{n+1} = 2^{n+2}$.

   The basis does not hold. Here, the basis is $s_0 = 1$ which is not equal to 2^2 as claimed.

5. (5 points) Consider the recursive definition: $a_0 = a_1 = 1$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 2$. Mark the claim that is true:

   (a) All terms a_n are odd (b) All terms a_n are even (c) There are two indices i, j such that a_i is odd and a_j is even.

   (a)

6. (5 points) The characteristic equation of the recurrence $s_n = 6s_{n-1} - 9s_{n-2}$ is (mark all that apply):

   (a) $x^3 - 6x^2 + 9x$ (b) $x^2 + 6x + 9$ (c) $x^2 - 6x + 9$ (d) $x(x-3)^2$ (e) $(x-3)^2$.

   (c), (e)