

Midterm Examination 1

October 20

1. **History**

- (a) Give one example of a symbol, other than $=$, that was used for equality.
- (b) In one sentence, state the difference between public key cryptography and private key cryptography.
- (c) Give the typical name used for two of the three participants (sender, receiver, eavesdropper) in secure communication. (Hint: think 1960's)
- (d) Give an example of Russell's paradox.

2. **Sets.** For each of the following relationships, state whether the relationship is true or false. If it is true, give a proof; if it is false, supply a counterexample. (Recall that \setminus is the relative complement operator.)

- (a) $A \setminus B = B \setminus A$.
- (b) $S \setminus (T \cup W) = (S \setminus T) \cap (S \setminus W)$.

3. **Functions.** Consider $f : R \times R \rightarrow R \times R \times R$ given by $f(x, y) = (y, x, x + y)$. Is f one-to-one? If so, prove it; if not, give a counterexample. Is f onto? If so, prove it; if not, give a counterexample.

4. **Logical Implication.** Prove that $(p \rightarrow q) \wedge \neg q \Rightarrow \neg p$.

5. **Proofs.**

- (a) Prove the following Lemma: if a^3 is even, then a is even.
- (b) Prove that $\sqrt[3]{2}$ is irrational. You may use the Lemma from part (a).