CS 1155 Understanding and Constructing Proofs **Fall Quarter, 1995**

Midterm Examination 2

November 17

1. **History**
   (a) What is the *sieve of Eratosthenes*?
   (b) Is it true that there exists a consecutive sequence of 521 integers, all of which are non-prime?
   (c) What is an Euler circuit?
   (d) In Euler's representation of the layout of Konigsberg as a graph, what physical entity is represented by an edge and what physical entity is represented by a vertex?

2. **Equivalence Relation.** On the set $N \times N$, define the relation $\sim$ by $(a, b) \sim (c, d)$ if $a + b = c + d$. State whether or not this is an equivalence relation. If your answer is "no", prove it. If your answer is "yes", prove it *and* sketch some of the equivalence classes on a 2-d plane.

3. **Relations.** Let R_1 and R_2 be binary relations on set S. For each of the following, either prove the expression is always true, or give a counterexample demonstrating that it can be false.
   (a) If R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is symmetric.
   (b) If R_1 is antisymmetric, then the relation R_2 defined by $\{(x, y) : x, y \in S, (x, y) \notin R_1\}$ is antisymmetric.

4. **More Relations.**
   Fill in the following table, indicating which properties hold for each of the three binary relations on the set $S = \{1, 2, 3\}$. (Fill in all boxes, putting a check mark if the property holds, and an "X" if it does not.)

Relations	Reflexive	Antireflexive	Symmetric	Antisymmetric	Transitive
$x = y$					
$\{(1, 2), (2, 1), (2, 3)\}$					
$\{(1, 2), (2, 1), (1, 1), (2, 2)\}$					

5. **Quantification.** For each of the following expressions, state whether it is true or not. Give an informal argument (this does not have to be a rigorous proof) about why your answer is correct. (Although the argument does not have to be a rigorous proof, it must be a convincing and accurate argument supporting your answer.)
   (a) $\forall x \in N\ \exists y \in N\ [x > 0 \rightarrow x + y \text{ even}]$
   (b) $\exists y \in N\ \forall x \in N\ [x > 0 \rightarrow x + y \text{ even}]$

(c) $\exists x \in N\ \forall y \in N\ [x > 0 \rightarrow x + y \text{ even}]$

6. **Big-Oh.** For $f, g : N \rightarrow N$ defined by $f(n) = 100n$ and $g(n) = n^2$ do the following:

   (a) Prove that $f(n)$ is $O(g(n))$.

   (b) Prove that $g(n)$ is not $O(f(n))$.