CS 1155 Understanding and Constructing Proofs **Fall Quarter, 1996**

Midterm Examination 1

November 4

1. NOR is a logic operator that is defined as follows: $p\ NOR\ q$ is true when both p and q are false; it is false otherwise.
   (a) Construct a truth table for NOR.
   (b) Show that $p\ NOR\ q$ is logically equivalent to $\neg(p \vee q)$.
   (c) Express $\neg p$ using only the NOR operator.

2. Is $(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$ a contradiction?

   If yes, prove your answer using a series of logical equivalences.

   If no, prove your answer by giving a counterexample to the claim that it is a contradiction.

3. For integers a and b, prove that if ab is odd then a is odd or b is odd.

4. Prove that the sum of a rational number and an irrational number is irrational. Use proof by contradiction.

5. State and prove a theorem that expresses exactly when $n^2 + 3$ is even, for natural numbers n.

6. Let the universe of discourse be the integers. Give the truth value and a brief explanation for parts (a) - (c);
   (a) $\forall x \exists y [x \neq 0 \rightarrow \frac{x}{y} = 1]$
   (b) $\forall y \exists x [x \neq 0 \rightarrow \frac{x}{y} = 1]$
   (c) $\exists y \forall x [x \neq 0 \rightarrow \frac{x}{y} = 1]$
   (d) Take the negation of the expression in part (a), simplifying as much as possible.

7. Let $P(x)$ denote "x is a professor"; $I(x)$ denote "x is idle"; and $V(x)$ denote "x is on vacation." Translate the following sentences into logic statements using quantifiers and logic operators.
   (a) No professors are idle.
   (b) All idle people are on vacation.
   (c) No professors are on vacation.
   (d) Does (c) follow from (a) and (b)? If yes, explain what rule(s) of inference are used. If no, explain why not.