CS 1155 Understanding and Constructing Proofs **Fall Quarter, 1997**

Midterm Examination 1

October 23

Throughout the exam:

- N denotes the natural numbers, $\{0, 1, 2, ...\}$
- R denotes the real numbers
- Z denotes the integers, $\{..., -2, -1, 0, 1, 2, ...\}$

1. Let $P(n)$ be the propositional function "n is a multiple of 4" and $Q(n)$ be the propositional function "n^3 is a multiple of 4", where n is a natural number. Consider the conjecture:

   For any natural number n, if n^3 is a multiple of 4, then n is a multiple of 4.

   (a) Write $P(n)$ and $Q(n)$ using only quantifiers, the symbol $=$ (which denotes the predicate "is equal to"), and the symbol $\times$ (which denotes multiplication).

   (b) Write the conjecture using only quantifiers, the propositional functions $P(n)$ and $Q(n)$, and the logic operators.

   (c) Write the negation of the conjecture using only quantifiers, the propositional functions $P(n)$ and $Q(n)$, and the logic operators. Simplify as much as possible.

   (d) Do one of the following two: (1) Prove the conjecture is true. (2) Prove the conjecture is false. State clearly which one you are doing.

2. Prove or disprove the following:

   (a) $\forall n \in N\, \exists m \in N[(m^2 \geq n) \rightarrow (m - n \text{ is odd})]$

   (b) $\exists m \in N\, \forall n \in N[(m^2 \geq n) \rightarrow (m - n \text{ is odd})]$

3. Prove that $((p \wedge q) \rightarrow r)$ is logically equivalent to $\neg(p \wedge q \wedge \neg r)$, using a series of logical equivalences. Give the name of the rule you are using at each step.

4. Prove that for all integers n, if $n^2 + n$ is odd, then n is even.

5. Consider the following statement: For all integers n and m, if nm is even, then n is even and m is even.

   (a) Give the English for the negation of the statement.

   (b) Prove or disprove the statement.

6. Let U be the set of all people who are, or once were, alive. Let the propositional function $A(x, y)$ denote that x is an ancestor of y, or in other words, y is a descendent of x.

   Use quantifiers, logic operators, and the propositional function $A(x, y)$ to express the following statements:

   (a) Everyone has at least one ancestor.

   (b) Any two people have a common ancestor.

   (c) All people have a common ancestor.

(d) Someone does not have descendants.
(e) Nobody is his or her own descendant.
(f) All descendants of Adam are descendants of Eve.
(g) Give the English and logic negations of the previous statement.

7. The NAND logic operator is false when both p and q are true; it is true otherwise.

   (a) Show that p NAND q is logically equivalent to $\neg(p \wedge q)$
   (b) Show that p NAND (q NAND r) is not logically equivalent to (p NAND q) NAND r.

8. Prove that the product of two rational numbers is rational.