

## Midterm Examination 2

November 20

Throughout the exam:

- $N$  denotes the natural numbers,  $\{0, 1, 2, \dots\}$
  - $R$  denotes the real numbers
  - $Z$  denotes the integers,  $\{\dots, -2, -1, 0, 1, 2, \dots\}$
1. Assume that  $A$ ,  $B$  and  $C$  are arbitrary sets with elements from some universe  $U$ . Using only the set identities in the table, and the additional identity  $A \cap \overline{A} = \emptyset$ , prove the following:
    - (a)  $\overline{A \cup (B \cap C)} = (\overline{A} \cap \overline{B}) \cup (\overline{A} \cap \overline{C})$
    - (b)  $(A \cup B) \cap \overline{(A \cap B)} = (A \cap \overline{B}) \cup (B \cap \overline{A})$
  2. Assume  $A$ ,  $B$  and  $C$  are sets with elements from the universe of natural numbers  $N$ . Prove or disprove the following for all sets  $A$ ,  $B$  and  $C$ :
    - (a)  $(A \subseteq B) \rightarrow ((A \cup C) \subseteq (B \cup C))$
    - (b)  $(A \subset B) \rightarrow ((A \cap C) \subset (B \cap C))$
  3. Let  $E = \{n \in Z \mid n \text{ is even}\}$  and  $A = \{n \in Z \mid \exists a, b \in Z, n = 6a + 8b\}$ . Prove that  $A = E$ .
  4. Let  $C = \{5, 10, 15\}$ ,  $f : C \rightarrow N$  where  $f(x) = x/3$ . For each of the following expressions, indicate whether the type of the expression is *set*, *proposition*, *function*, or *ill-formed*:
    - (a)  $\emptyset \in C$
    - (b)  $C \times C$
    - (c)  $5 \cup 10$
    - (d)  $f$  is one-to-one
    - (e)  $C \vee C$
  5. Prove that the function  $f : R \rightarrow R$  defined by  $f(x) = 3x - 4$  is one-to-one and onto.
  6. Prove or disprove that the function  $f : Z \rightarrow Z$  defined by  $f(x) = x^2 + 5x + 6$  is one-to-one.
  7. Prove that  $\sqrt{3}$  is irrational.