CS 1155 Understanding and Constructing Proofs **Summer Quarter, 1996**

Midterm Examination 1

August 30

1. Using truth tables, verify whether or not the following statements are tautologies:
   (a) $\neg(p \oplus q) \leftrightarrow (p \leftrightarrow q)$
   (b) $(p \rightarrow q) \wedge \neg p \rightarrow \neg q$

2. Is $p \rightarrow q \leftrightarrow p \wedge \neg q$ a contradiction?

   If yes, prove your answer using a series of logical equivalences.

   If no, prove your answer by giving a counterexample to the claim that it is a contradiction.

3. Consider the universe U of all students. Let $P(x)$ denote "student x gets up early."
   (a) Express the proposition "Not all students get up early" using quantifiers and the predicate P.
   (b) Express the proposition "All students do not get up early" using quantifiers and the predicate P.
   (c) Let Q denote the proposition from part (a) and R denote the proposition from part (b). Does $Q \Rightarrow R$? Does $R \Rightarrow Q$? Argue why your answers are correct.

4. Let $E(m, n)$ be the predicate $m \neq n$ for universes of discourse N, the natural numbers. Give the truth value of each of the following, with a counterexample for those that are false:
   (a) $\forall n \exists m E(m, n) \rightarrow \exists m \forall n E(m, n)$
   (b) $\neg \forall m \exists n E(m, n) \rightarrow \neg \exists n \forall m E(m, n)$

5. Recall from the homework that the notation $\exists! x P(x)$ denotes the proposition "There exists a unique x such that P(x) is true."

   Give the truth values of the following statements, and a counterexample if the statement is false.
   (a) $\exists! x P(x) \rightarrow \exists x P(x)$
   (b) $\exists! x \neg P(x) \rightarrow \neg \forall x P(x)$

6. Let $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$. For each of the following expressions, indicate whether the type of the expression is *set*, *proposition*, *assertion about propositions* or *ill-formed*:
   (a) $A \subseteq B$
   (b) $A \cup B$
   (c) $A \subset B \leftrightarrow A \subseteq B \wedge A \neq B$
   (d) $A \subset B \Leftrightarrow A \subseteq B \wedge A \neq B$
   (e) $A \wedge B$

7. (a) Give an example of sets A and B that demonstrate that the relative complement is *not* commutative, i.e., $A \setminus B \neq B \setminus A$.

   (b) Give an example of sets A and B for which the relative complement *is* commutative, i.e., $A \setminus B = B \setminus A$.

8. Suppose that A, B and C are sets such that $A \oplus C = B \oplus C$. Does this imply that $A = B$?

9. Prove that if A and B are sets, then $(A \cap B) \cup (A \cap B^c) = A$.