CS 1155 Understanding and Constructing Proofs	**Summer Quarter, 1996**

Midterm Examination 2

September 20

Throughout the exam, the symbol $\mathbb{N}$ denotes the natural numbers, and the symbol $\mathbb{R}$ denotes the real numbers.

1. **Functions.** Consider $f : \mathbb{N} \to \mathbb{N}$ given by $f(n) = |n - 2|$, where $|x|$ denotes the absolute value of x.
   (a) Is f one-to-one? If yes, give a proof; if no, give a counterexample.
   (b) Is f onto? If yes, give a proof; if no, give a counterexample.
2. **Functions.** Consider $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ given by $f(a, b) = (b, 2a, a + b)$.
   (a) Is f one-to-one? If yes, give a proof; if no, give a counterexample.
   (b) Is f onto? If yes, give a proof; if no, give a counterexample.
3. **Function Composition.** Let $f(x) = ax + b$ and $g(x) = cx + d$. What are the conditions on the constants a, b, c and d so that $f \circ g = g \circ f$? Justify your answer.
4. **Types.** Let $A = \{1, 2, 3\}$, $f : A \to A$ where $f(x) = x$, and $R = A \times A$. For each of the following expressions, indicate whether the type of the expression is *set*, *proposition*, *function*, *assertion about propositions* or *ill-formed*:
   (a) f is onto
   (b) $A \subseteq R$
   (c) $f(1) \cup f(2)$
   (d) $\{\emptyset\}$
   (e) $f = O(1)$
5. **Big-Oh.** Let f and g be functions from $\mathbb{N}$ to $\mathbb{N}$ where $f(n) = 4n^2 + n + 5$ and $g(n) = n^3$. Prove that $f(n)$ is $O(g(n))$, using the logic-based definition of big-Oh.
6. **Big-Oh.** Let f and g be functions from $\mathbb{N}$ to $\mathbb{N}$ where $f(n) = n^3$ and $g(n) = 100n^2$. Is $f(n) = O(g(n))$? Prove that your answer is correct, using the logic-based definition of big-Oh.
7. **Big-On.** Let f, g and h be arbitrary functions from $\mathbb{N}$ to $\mathbb{N}$. Prove that if $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$, then $f(n)$ is $O(h(n))$. Use the logic-based definition of Big-Oh.
8. **Relations.** Suppose that R and S are relations on a set A. Prove or disprove the following:
   (a) R is reflexive and S is reflexive implies $R \cup S$ is reflexive.
   (b) R is symmetric and S is symmetric implies $R \cup S$ is symmetric.
9. **Relations.** Let $A = \{a, b, c\}$.
   (a) Give an example of a relation on A that is antisymmetric **and** transitive, but **not** reflexive.
   (b) Give an example of a relation on A that is **neither** symmetric **nor** antisymmetric.