

Every assignment will be due at the beginning of class. Recall that you can collaborate in groups and/or use external references, but you must acknowledge the group/references that you used, and you must *always write your solutions alone*. Remember that for 90% of the people, more than 50% of the understanding happens during writing/implementation/etc. (And this is not true only for CS 1050. It is true for mostly everything, at least technical).

Please read the entire homework before starting to work on it. This homework refers to material covered in Lectures II through V. You may want to refer to the readings assigned during these Lectures, and to the Lecture Outlines. All are posted on the class website.

Please stop by for questions during office hours of instructor or TAs and send email to mihail@cc.gatech.edu with title 1050 at any time. This helps you, but it also helps us! Sometimes it helps us understand where the class stands and where we should put more or less emphasis. And sometimes, you give us presentational and technical ideas that we would have not thought of otherwise. So keep all communication links open!

Please print this document, and write your solutions on the printout.  
Please hand-in the completed printout.

PRINT YOUR NAME HERE:.....

WRITE YOUR EMAIL HERE:.....

### Problem 1: (10 points)

Which of the following statements are propositions, and which are not propositions. For each statement that is a proposition, write (without explanation) if it is true or false. For each statement that is not a proposition, give a one line explanation why it is not a proposition.

(i)  $2+2 = 4$

(ii)  $2+7 = 10$

(iii)  $\forall x \forall y (x + y = y + x)$ , where  $x$  and  $y$  are real numbers.

(iv)  $\forall x \exists y (x + y = y + x)$ , where  $x$  and  $y$  are real numbers.

(v) If the earth is flat, then  $2+2=7$ .

(vi) Go directly to jail.

(vii) Your place or mine?

(viii)  $3 = x + 7$ .

(ix) If  $2+2=4$  then  $2+3=5$ .

(x) If  $2+2=5$  then  $2+3=6$ .

## Problem 2: (10 points)

The statement  $x - y = y - x$ , where  $x$  and  $y$  are real numbers is not a proposition. This is because its truth value depends on  $x$  and  $y$ . For example, if  $x = y = 0$  then  $x - y = y - x$ , while if  $x \neq 0 \neq y$  then  $x - y \neq y - x$ . To turn the statement  $x - y = y - x$  to a proposition, we have to bind the variables  $x$  and  $y$  by existential and/or universal quantifiers. For each of the four ways of binding  $x$  and  $y$  that follow, state if the corresponding proposition is true or false. Give a short explanation. If you find some cases somewhat hard or unclear, try to do Problem 3 (or part of it) first.

(i)  $\forall x \forall y (x - y = y - x)$ .

(ii)  $\forall x \exists y (x - y = y - x)$ .

(iii)  $\exists x \forall y (x - y = y - x)$

(iv)  $\exists x \exists y (x - y = y - x)$

### Problem 3: (10 points)

In Problem 3, statements (i) through (iv) are propositions, Therefore, their negations are also propositions. For each of the propositions (i) through (iv) in Problem 3, write clearly its negation, where the negation sign  $\neg$  does not appear anywhere in the proposition.

For example, for (i), the negation of  $\forall x \forall y (x - y = y - x)$  is  $\neg[\forall x \forall y (x - y = y - x)]$ .

This is equivalent to  $\exists x \neg[\forall y (x - y = y - x)]$ .

This is equivalent to  $\exists x \exists y \neg(x - y = y - x)$ .

This is equivalent to  $\exists x \exists y (x - y \neq y - x)$ .

You may even go one step further, and observe that  $(x - y \neq y - x) \Leftrightarrow 2x \neq 2y \Leftrightarrow x \neq y$ .

You then get the cleanest form:  $\exists x \exists y x \neq y$ .

### Problem 4: (10 points)

Using truth tables, prove the following tautologies (where  $p$  and  $q$  are propositional variables):

(i)  $[p \wedge (p \Rightarrow q)] \Rightarrow q$  (aka Modus Ponens or Direct Proof).

(ii)  $[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$  (aka Modus Tollens or Contradiction).

(iii)  $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$  (aka Contraposition).

(iv)  $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$ .

### Problem 5: (10 points)

Express these systems specifications using the propositions  $p$  "The message is scanned for viruses" and  $q$  "The message was sent by an unknown system" together with logical connectives.

(a) "The message is scanned for viruses whenever the message was sent from an unknown system".

(b) "The message was sent by an unknown system but it was not scanned for viruses".

(c) "It is necessary to scan the message for viruses whenever it was sent from an unknown system".

(d) "When a message is not sent from an unknown system it is not scanned for viruses".

Show that the above specifications are not consistent, and explain it also in English. Then show that if you drop exactly one of (a), (b), (c), or (d), then the system will become consistent, thus completely identifying the cause of inconsistency.

### Problem 6: (10 points)

A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the cook and the gardener are not both lying; and if the naahdyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.

Problem 7: (10 points)

(iA) Give a direct proof (one line argument) that the product of two consecutive integers is always an even number.

(iB) Give a direct proof that, for any integer  $k$ ,  $k^2 + k$  and  $k^2 - k$  are both even numbers.

(iiA) Prove that, if  $n$  and  $m$  are odd integers, then the product  $n \times m$  is also an odd integer. A simple direct proof should suffice.

(iiB) Prove that, if  $n$  and  $m$  are integers and the product  $n \times m$  is an odd integer, then both  $n$  and  $m$  are odd integers. You may need to use the contrapositive principle for this proof.

Problem 8: (10 points)

Prove that  $\sqrt{3}$  is an irrational number. You may need to use an argument by contradiction.

### Problem 9: (10 points)

(i) What is wrong with the following argument:

If  $\sqrt{2} > \frac{3}{2}$  then  $(\sqrt{2})^2 > \left(\frac{3}{2}\right)^2$ .

But we know that  $\sqrt{2} > \frac{3}{2}$ ,

therefore  $(\sqrt{2})^2 = 2 > \left(\frac{3}{2}\right)^2 = \frac{9}{4}$ .

(ii) What is wrong with the proof of the following theorem:

Theorem: If  $n$  is an integer and  $3n + 10$  is odd then  $n$  is odd.

Proof:  $n$  is odd,

therefore  $3n$  is odd (because the product of two odd numbers is always odd)

therefore  $3n + 10$  is odd (because the sum of an odd and an even integer is always odd).

## Problem 10: (10 points)

Prove that every odd integer is the difference of two squares.

Hint: The proof is direct and very simple. But how does one start thinking about it?

Examples is many times a good place to start, since they might indicate a pattern. In this case, try to express 1, 3, 5, 7, 9 as the difference of two squares, and you immediately see the pattern. For example, for 3, it is obvious that  $3 = 4 - 1 = 2^2 - 1^2$ . Also 5 is easy:  $5 = 9 - 4 = 3^2 - 2^2$ . Now keep going...

### Problem 11, (10 points, Extra Credit)

Four friends have been identified as suspects to an unauthorized computer system. They have made statements to the investigating authorities. Alice said "Carlos did it". John said "I did not do it". Carlos said "Diana did it". Diana said "Carlos lied when he said that I did it".

(a) If the authorities also know that exactly one of the suspects is telling the truth, who did it? Explain your reasoning.

(b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.

## Problem 12, (10 points, Extra Credit)

Consider the standard  $8 \times 8$  checkboard, with its squares colored alternatively black and white in the standard way. Prove that you can remove any black square and any white square, and still be able to tile the remaining part of the checkboard with  $2 \times 1$  tiles.

Hint 1: Try several examples, and realize that, after removing any pair of one black and one white square, you can always partition the remaining board, so that each partition class can be tiled with  $2 \times 1$  tiles.

Hint 2: Look at the figure referring to exercise 44 on page 104 of Rosen's book. This figure suggests an "E-like" structure embedded in the checkboard. This structure can also help you partition the remaining board, so that tiling is always possible. But can you explain how one would come up with this structure? If you follow this route, can you explain how one could have come up with this "E-like" structure in the first place?