11-17-08 Homework 6
CS 1050 A Due: 11-25-08

Every assignment will be due at the beginning of class. Recall that you can collaborate in groups and/or use external references, but you must acknowledge the group/references that you used, and you must *always write your solutions alone.* Remember that for 90% of the people, more than 50% of the understanding happens during writing/implementation/etc. (And this is not true only for CS 1050. It is true for mostly everything, at least technical).

Please read the entire homework before starting to work on it. The first 5 questions are on order of growth, and elementary probability/statistics, as discussed in class. Question 5 is a revision of basic proof techniques that you should have solid for Quiz 3 and the Final Exam.

Please stop by for questions during office hours of instructor or TAs and send email to mihail@cc.gatech.edu with title 1050 at any time. This helps you, but it also helps us! Sometimes it helps us understand where the class stands and where we should put more or less emphasis. And sometimes, you give us presentational and technical ideas that we would have not thought of otherwise. So keep all communication links open!

Please print this document, and write your solutions on the printout.
Please hand-in the completed printout.

PRINT YOUR NAME HERE:... ....

WRITE YOUR EMAIL HERE:.. ....

Problem 1: Growth of functions (20 points)

Sort the following functions according to order of growth, sfrom smaller to larger. No explanation is needed.

$2^n,\ n\sqrt{\log n},\ n^3,\ n\sqrt{n},\ \log n,\ n\log n,\ \sqrt{\log n},\ \sqrt{n},\ 2^{n^2}.$
Make sure you read the Lecture Outlines (I) and (II) of Tue Nov. 4, before answering this question.

Problem 2: $O()$ notation (20)

(a) Prove that $\sqrt{n}\log n = O(n)$. Make sure you read the Lecture Outlines (I) and (II) of Tue Nov. 4, before answering this question. You should answer this question in the way similar to the examples of Lecture Outline (II) of Tue Nov. 4.

(b) Prove that $n^3 + 10n^2 + 1,000,000 = O(n^3)$. Make sure you read the Lecture Outlines (I) and (II) of Tue Nov. 4, before answering this question. You should answer this question in the way similar to the examples of Lecture Outline (II) of Tue Nov. 4.

Problem 3: Averages (10)

In a test of a class of 100 students the average score was 90, and all the scores were between 0 and 100.

(a)Could it be that 50% of the students got a score of 60 or less? Fully justify your answer. Make sure you read the Lecture Outline of Thu Nov. 6, before answering this question. You should answer this question in the way similar to the examples of Lecture Outline of Thu Nov. 6.

(b) Suppose that 10 students' score was 0 (they did not show up). What can you infer about the scores of the other 90 students? Fully justify your answer. Make sure you read the Lecture Outline of Thu Nov. 6, before answering this question. You should answer this question in the way similar to the examples of Lecture Outline of Thu Nov. 6.

Problem 4: Averages (10)

uppose that the average income of a population of 1M people is 100K, and 99.5% of the population makes no more than 10K.
(a) Could it be that the population contains no millionnaire? Make sure you read the Lecture Outline of Thu Nov. 6, before answering this question. You should answer this question in the way similar to the examples of Lecture Outline of Thu Nov. 6.

(b) What is the average income of the remaining 0.5% of the population? Fully justify your answer. Make sure you read the Lecture Outline of Thu Nov. 6, before answering this question. You should answer this question in the way similar to the examples of Lecture Outline of Thu Nov. 6.

Problem 5: Averages in programming, and Order of growth (20)

A computer program takes as input a 0-1 bit string of length n, and outputs a nice picture.
(a) Argue that, for each n, there are 2^n possible inputs to the program.
(b) Suppose that, for each $n \geq 1$, on all but 10 special inputs the program produces the picture in n^2 milliseconds. On each one of the 10 special inputs the program takes 2^n milliseconds. Let $t(n)$ be the average running time of the program, over all inputs of 0-1 bit strings of length n. Compute $t(n)$ as a function of n.
(c) Prove that $t(n) = O(n^2)$.

(d) Now suppose that, for each $n \geq 1$, for 10% of the 2^n inputs the program takes 2^n milliseconds, and for the remaining n^2 of the inputs the program takes n^2 milliseconds. Let $T(n)$ be the average running time of the program, over all inputs of 0-1 bit strings of length n. Compute $T(n)$ as a function of n.

(e) Prove that $T(n) = O(2^n)$.

Problem 6: Revision: Direct, Indirect, Inductive Proofs (20)

(a) In your messy drawer you have 10 pairs of red socks and 10 pairs of blue socks. It is 5:30 am, it is pitch black, your lightbulb is broken, you cannot tell red from blue. You want to pick a bunch of socks, so that once you get downstairs and switch some light on, you have at least two socks of the same color. What is the minimum number of socks that you have to pick to be sure that you will have atleast two socks of the same color. Justify your answer (there is a 2 line direct argument).

(b) Prove that $\sqrt{7}$ is irrational. State clearly which proof technique you are using.

(c) Prove that, if for some integer n, the number $n^4 - 51$ is even, then n must be odd. State clearly which proof technique you are using.

(d) Prove, by induction, that the solution to the recurrence $f(n) = f(n-1) + n$ with $f(1) = 1$ is $f(n) = n(n+1)/2$.