

This problem-set on the primal-dual schema for approximation algorithms. Before you start working on this problem-set, make sure you have read and understood the following material:

- (a) Vazirani's book, chapter 15, and in particular paragraph 15.1, which is an overview of the primal-dual schema with relaxed complementary slackness conditions.
- (b) Vazirani's book, chapter 22, which is the Steiner Forest problem, and exercises 22.7, 22.8, 22.9 and 22.12 (for the exercises, you do not have to know how to solve them; you just have to know which problems they refer to, and that these problems can be solved by similar methods as the Steiner Forest problem).
- (c) Vazirani's book, chapter 24, and all the exercises in paragraph 24.5 (again, for the exercises, you do not have to know how to solve them; you just have to know which problems they refer to, and that these problems can be solved by similar methods).

Problem 1 (easy)

- (a) Answer exercise 22.2, page 206, in Vazirani's book. In addition to the hint given in the book, look at Theorem 3.2, page 27, in Vazirani's book.
- (b) Answer exercise 22.4, page 206, in Vazirani's book.
- (c) Look at the definition of a *proper function* in exercise 22.7, page 207, in Vazirani's book. Prove that the function f defined by the Steiner Forest Problem 22.1, page 197, is indeed a proper function.

Problem 2 (regular)

Answer exercise 22.10, page 209, in Vazirani's book. Hint: For all vertices i and j , write the requirement r_{ij} between vertices i and j in binary (using at most $\lfloor \log_2 k \rfloor + 1$ bits), and call Algorithm 22.3 $\lfloor \log_2 k \rfloor + 1$ times as a subroutine.

Problem 3 (easy)

- (a) Consider the general uncapacitated facility location problem in which the connection costs are not required to satisfy triangle inequality. Give a reduction from the set cover problem to show that approximating this problem is as hard as approximating set cover.
- (b) Answer exercise 24.6, page 239, in Vazirani's book.

Problem 4 (regular)

Answer exercise 24.7, page 239, in Vazirani's book.