

This is a short problem set. This problem set is on  $k$ -median and the method of Lagrangian relaxation (lectures Feb 24 and March 4), and on randomization, derandomization and randomized rounding, which was used for both the  $k$ -median factor 6 approximation algorithm (lecture March 4) and the MAXSAT factor 3/4 approximation algorithm (lecture March 6). Please make sure you understand Vazirani's chapters 25 and 16 before answering the problems on this homework.

### Problem 1 (above average but not hard)

(Exercise 25.3 from Vazirani, page 250). Use the Lagrangian relaxation technique to give a constant factor approximation algorithm for the following common generalization of the facility location and  $k$ -median problems. Consider the metric uncapacitated facility location problem with the additional constraint that at most  $k$  facilities can be opened. Assume for simplicity that the number of cities is equal to the number of facilities, say  $n$ . If you find it hard to solve the problem to the end, try to carry the techniques of chapter 25 as far as you can, and explain how and why you cannot proceed further.

### Problem 2 (regular-easy, mostly easy)

(Exercise 16.8 from Vazirani, page 137). MAX  $k$ -CUT,  $k \geq 2$ , is the following problem: Given an undirected graph  $G(V, E)$  with nonnegative edge costs, and an integer  $k$ , find a partition of  $V$  into sets  $S_1, \dots, S_k$  so that the total cost of edges running between these sets is maximized. Consider the obvious randomized algorithm for the MAX  $k$ -CUT problem, which assigns each vertex randomly to one of the sets  $S_1, \dots, S_k$ . Show that the expected cost of edges running between these sets is at least  $\text{OPT}/2$ . Explain how to derandomize this algorithm, for a deterministic algorithm with performance guarantee at least  $\text{OPT}/2$ .

Extra Credit (really not mandatory): If you are not challenged enough by the above problems, try (a) Exercise 25.4/Problem 25.6 (Metric  $k$ -MST) in page 251 of Vazirani's book, and/or (b) Exercise 25.6 page 252 in Vazirani's book (clustering for 2-norm).