CS 4540, Advanced Algorithms
Homework 2

Fri, Sept 10, 2010 Due Fri, Sept 17, 2010

Problem 1

Motwani and Raghavan, Problem 4.1, page 97. Note: The purpose of this problem is to familiarize you with the use of Chernoff bounds. You may use any of the foffowing forms of Theorems 1 through 5 given at the Class Notes of 09-08-10 (see Class Notes on the class web site).

Problem 2

Let $0 \leq y_1 \leq y_2 \leq \ldots \leq y_{2N} \leq 1$ be $2N$ real numbers, with $y_{2N+1-j} = 1 - y_j$, for $1 \leq j \leq N$. Let $X_1, X_2, \ldots, X_n$ be independent random variables such that, for $1 \leq i \leq n$ and for $1 \leq j \leq N$, $\Pr[X_i = y_j] = p_{ij}$ and $\Pr[X_i = y_{2N+1-j}] = \Pr[X_i = y_j]$. Let $p_i = E[X_i] = \sum_{j=1}^{2N} y_j p_{ij}$, where $0 < p_i < 1$. Finally, let $X = \sum_{i=1}^{n} X_n$ and let $\mu = E[X]$.
(a) Prove that, for any $\delta > 0$, $\Pr[X > (1+\delta)\mu] < \left[\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right]^{\mu}$.
(b) Prove that, for any $1 > \delta > 0$, $\Pr[X < (1-\delta)\mu] < \left[\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right]^{\mu}$.
Note: The purpose of this problem is to familiarize you with the proof of the basic form of Chernoff bounds. You may use as guideline the proof of (13.42) in page 749 of Kleinberg and Tardos, and/or the proofs of Theorems 4.1 and 4.2 in pages 68 and 70 (respectively) of Motwani and Raghavan. (You may also use any other guideline you wish).

Problem 3

(a) Exercise 2, page 782 of Kleinberg and Tardos.
(b) Consider the same scenario as in Exercise 2, page 782 of Kleinber and Tardos of part (a), with the following exception: The probability that each voter votes for the wrong candidate is not $\frac{1}{100}$, but some unknown ϵ, where $0 \leq \epsilon \leq 1$. What is the largest value of ϵ that you would recommend to the Decocratic party to tolerate, so that, with probability at least 99.9% their candidate will end up with at least 79,000 votes?
(c) Suppose again that there are 100,000 voters, however, the voter sentiment is reversed. 20,000 voters intend to vote for the Democratic candidate, while 80,000 voters intend to vote for the Republican candidate. Suppose again that the ballot layout is quite confusing, so that, each voter, independently and with probability ϵ, where $0 \leq \epsilon \leq 1$, votes for the wrong party. Determine a value of ϵ for which the Democratic canditate has at least 1% chance of winning this election.

Problem 4

(a) Exercise 4, page 784 of Kleinberg and Tardos, *only part* (a), *ie, do not do part* (b) *on page 789.*
(b) For the exact same scenario as in Exercise 4, page 784 of Kleinberg and Tardos, determine a constant c such that you can prove that the probability that there exists a vertex with more than $c \ln n$ incoming links is at most $\frac{1}{n}$.