A Re-dial Model for Personal Communications Services Networks

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Abstract - This paper studies the effect of re-dial in a personal communications services (PCS) network. The re-dialing problem for PCS is different from the plain old telephone service (POTS). When using POTS, if a customer receives a busy signal in making a call, or the call does not connect in a timely manner, the customer may make a re-dial call in an attempt to get a connection. Likewise, it is anticipated *personal communications services* (PCS) network subscribers may also make re-dial attempts when the original call attempt is unsuccessful. However, unlike POTS, PCS has the added complexity of keeping a call connected while the originating caller and/or the terminating caller is moving. Because of this complexity, the effect of re-dial on nomadic communications is a challenging problem that has implications on the design and analysis of PCS networks. Accordingly, this paper presents a model for studying the effect of re-dial on PCS networks. Using this model, several analytical results are presented. These results are then used to validate a Time Warp based, distributed, simulation of the PCS re-dial model. Using the distributed simulation, we vary (i) the probability of re-dial, (ii) the re-dial rate, and (iii) portable mobility and examine each parameter's effect on (a) the probability of forced termination, (b) the probability of a new call block, (c) the call incompletion probability, (d) expected number of re-dials before a new call attempt is accepted or rejected.

I Introduction

This paper studies the effect of re-dialing in a *personal communications services* (PCS) network. In a PCS network, the service area is populated with a large number of radio ports with each providing coverage in its vicinity. We consider a fixed or quasi-static channel assignment [4] where a group of channels (time slots, frequencies, spreading codes or a combination of these) are assigned to each port.

When a call arrives at the *coverage area* of a port, the destination (or the originating) portable is connected if a channel is available. Otherwise, the call is blocked. When a communicating portable moves from one coverage area to another, the channel in the old port is released, and a channel is required in the new port. This process is called a *hand-off*. If no channel is available in the new port, then the hand-off call is said to be *forced terminated*.

When a new call attempt is blocked, the caller may re-dial after a short period. After several re-redials, either the call is connected, or the caller decides to drop the call attempt. We define the *new call blocking probability* as the probability that a new call attempt is dropped after some number of re-dialings. When a new call arrives, the radio port first checks if a channel is available to accommodate the call. If so, the call is connected until the conversation is completed or the corresponding portable moves out of the coverage area (and a hand-off occurs). If no channel is available, the caller determines if a re-dial will occur. If not, the call is blocked. Otherwise, the caller re-dials after a short period. If the portable moves out of the coverage area before the re-dial, then the re-dial call occurs at the destination coverage area. For a hand-off call arrival, if an idle channel is available, then the call is connected. Otherwise the call is forced terminated.

We study the effect of re-dialing on PCS network performance. The paper is organized as follows. Section II derives several output measures analytically. Section III describes a distributed simulation testbed that is based on Jefferson's *Time Warp* synchronization mechanism. In Section IV, the analytic results are used to validate the simulation technique described in the previous section. We then discuss the impact of re-dialing on a PCS network based on the experimental results. Section V presents the conclusions of this study and future work.

II Analysis

This section derives several output measures for an exponential portable residual time distribution. The equations derived will be used to validate the simulation model in the next section (the exponential assumptions can be easily generalized in the simulation model). Throughout this study, the following assumptions and notations are used.

- The call arrivals to a portable are a Poisson process.
- The portable residual times in a coverage area are exponentially distributed with the mean $1/\eta$ as suggested in [10].
- The call holding time is exponentially distributed with the mean $1/\mu$.
- The interval between two re-dials (for the same portable) is exponentially distributed with the mean $1/\gamma$.

- α is the probability that a portable (or the caller to the portable) decides to re-dial after a call attempt failure.
- β is the probability that no channel is available when a call attempt (either a new call or a hand-off call) arrives.
- p_f is the forced termination probability (the probability that a hand-off call is blocked).
- p_o is the probability that a new call is blocked (after some re-dials without success).
- p_{nc} is the call incompletion probability. This probability includes both new call blocks and hand-off calls that are forced to terminate. An incomplete call is either blocked as a new call or it may make several successful hand-offs before it is forced terminated.

The call incompletion probability p_{nc} for general portable residual time distribution was derived in [8, 9]. A simple alternative derivation of p_{nc} for the exponential residual time distribution is found in [3]. The final result is as follows:

$$p_{nc} = \frac{\mu p_o + \eta p_f}{\mu + \eta p_f} \tag{1}$$

The expected number N_r of re-dials (excluding the first dial) before a new call attempt is accepted or rejected is derived in [3]. The final result is as follows.

$$N_r = \frac{\alpha\beta}{1 - \alpha\beta} \tag{2}$$

The probability p_o is also derived in [3]. The final result is:

$$p_o = \frac{(1 - \alpha)\beta}{1 - \alpha\beta} \tag{3}$$

Let C_r be the expected number of idle channels observed by a re-dial call. An upper bound, C_r^+, of C_r can be derived by assuming that no new calls, hand-off calls and other re-dial calls have arrived between two re-dials by the same user. A detailed derivation is contained in [3]. This final result from that derivation is:

$$C_r^+ = \sum_{0<i<\infty} i \binom{c}{i} \theta^i (1 - \theta)^{c-i} \tag{4}$$

III The Parallel Simulation Technique

Based on our experience in PCS Radio Port level simulations, good confidence intervals in the output statistics can only be obtained when the length of time simulated is very long [2]. For example, to simulate just call arrivals with re-dial capability and call hand-offs at the Radio Port, the number of events processed in each run is between 10,000,000 and 100,000,000. Because of the large number events that need to be processed, execution times for the simulation are very high.

In an attempt to solve the large execution time problem in other large simulation applications, a number of workstations have been used simultaneously, where each workstation executes a copy of the simulation with different parameter settings. The problem with this approach is that it still requires a significant amount of time to complete a batch of simultaneous runs. Moreover, the parameter settings for a simulation run can depend on the results generated from prior runs. Accordingly, if a "stab in dark" or "best guess" approach is taken to the setting of simulation parameters, many of the runs in a batch may be of no value because of poorly chosen parameter settings. Because there are many simulation parameters in the PCS re-dial simulation model, we believe a "batch" execution approach will not provide the necessary time efficiency to effectively analyze this wireless network.

To reduce the execution time of the PCS simulation, we use an optimistic parallel/distributed simulation method known as Time Warp [7]. The Time Warp mechanism uses a detection-and-recovery protocol to synchronize the computation. For a more detailed discussion of Time Warp and other parallel simulation techniques, we refer the reader to [7, 5, 6].

A version of Time Warp has been developed that executes on a collection of DEC 5000 workstations, Sun Sparc workstations, or a mixture of these machines. The Time Warp system is written in C++. A principal objective of this implementation is to enable efficient simulation of thousands of "light weight" LPs (i.e., processes that contain a small amount of state and perform little computation in each event) in an object-oriented environment on networked, heterogeneous workstations. A more detailed description of this Time Warp implementation is provided in [2, 1].

While the performance of the simulation is not the primary focus of this paper we feel it should be briefly mentioned. The results presented in this study are the product of over 150 executions of the distributed simulator. Each run simulates over $5,000,000$ new call attempts. The average time required to execute a single run of the distributed simulator was less than 13 minutes using 8 workstations. Had only a single workstation been used, it would have taken almost 1.5 hours to complete a simulation run. For a more detailed examination of PCS network performance under Time Warp, the reader is referred to [2].

IV Discussion

In this section, we first validate our simulation model against the analytic equations for p_o, p_{nc}, and N_r and present the simulation results concerning the impact of re-dialing on a PCS network.

All executions were conducted on 8 DEC 5000 workstations, with each processor containing 128 distinct *coverage areas*, for a total of 1024 *coverage areas*. The simulated call holding time for a phone call is exponentially distributed with mean of $1/\mu = 3$ minutes. The number of channels per *coverage area* is fixed at 8. All executions ran for 2.5×10^5 simulated seconds.

To validate our simulation, we compared the predicted values for p_o, p_{nc}, and N_r, which are based on the equations in Section II, with values produced by the simulation. For these comparisons, we varied (i) the probability of re-dial, α, (ii) the re-dial rate, γ, and (iii) the portable mobility, η. We grouped the parameters into three sets: *low, medium and high*. The *low* set consists of $\alpha = 0.1, \gamma = 0.1\mu$, and $\eta = 0.04\mu$. The *medium* set consists of $\alpha = 0.5, \gamma = 1.0\mu$, and $\eta = 0.2\mu$.

Last, the *high* set consists of $\alpha = 0.9$, $\gamma = 10.0\mu$, and $\eta = 1.0\mu$. Additionally, a final comparison is made based the upper bound for the expected number of idle channels seen by a re-dial call, C_r^+. Here, γ is varied for a fixed η. In all cases, the simulation tracks the analytically derived values for p_o, p_{nc}, N_r and C_r^+. Due to space limitations, these results are not presented here, but are presented in [3].

To study the effect of re-dials on PCS networks, we vary (i) α, the probability of re-dial, (ii) γ, the re-dial rate, and (iii) η, portable mobility and examined the effect of each parameter on (a) p_f, the probability of forced termination, (b) p_o, the probability of new call block, (c) p_{nc}, the call incompletion probability, (d) N_r, the expected number of re-dials before a new call attempt is accepted or rejected.

In Figure 1, the effect of α on p_f, p_o, p_{nc} and N_r as a function of the offered load is shown. It can be seen from Figure 1(a), that p_f increases for larger values of α, especially when the offered load is high (greater than 3.5 Erlangs). These results are attributed to (i) increasing the offered load causes a decrease in new call interarrival times, which leads to fewer available channels to hand-off calls and (ii) increasing α increases the number of active re-dial calls vying for an available channel, which also leads to an increase in the number of forced terminations.

In Figure 1(b), α has a opposite effect on on p_o. As the re-dial probability is increased, a new call attempt is given more opportunities to connect before it is counted as a blocked call. Accordingly, the new call blocking probability will be much lower for high values of α, especially at higher offered loads, where the observed channel availability is less.

In Figure 1(c), α has the same effect on p_{nc} as on p_o. The reason for this similarity is because the number of new call attempts (the basis used to compute p_o) is much larger than the number of hand-off call attempts (the basis used to compute p_f). Accordingly, p_o has a large influence on p_{nc} for lower η values. However, when η is large, p_o looses its influence on p_{nc}. This effect will be shown when η is varied.

In Figure 1(d), the expect number of re-dials made by a call attempt increases as α is increased. This result follows one's intuition. For a given fixed load, as the probability for re-dial is increased, the number of re-dials made by a call will increase, thus increasing N_r.

The effect of γ is shown in Figure 2. It is observed in figures 2(a) and (b) that while p_f is insensitive to changes in γ while p_o is affected as γ is varried. While intuitively, one would expect to see an inverse relationship between these two probabilities (as p_o increases, p_f should decrease and vice versa), this however, does not appear to be the case. As γ is increased, the probability that a channel will become free is decreased. In the 10μ case (the re-dial rate is 10 time faster the call hold rate) a call would need to make 10 re-dial attempts on average before a channel would become available, which has a probability of less than 0.35 (assuming $\alpha = 0.9$). Because the observed channel availability is much lower at higher γ values, p_o is increased. Now, when p_o increases, the arrival rates for first time hand-offs calls decreases and the availability of channels for these hand-off calls increases. The net effect is that p_f should decrease. However, when p_f decreases, the arrival rates for K-ary, (where $K > 1$), hand-offs increases, resulting in an increase in p_f. Accordingly, a change in p_f is not observed.

The effect of γ on p_{nc} (Figure 2(c)) is again due to the large influence p_o has on p_{nc} for a fixed η.

N_r also increases as γ increases, as shown in Figure 2(d).

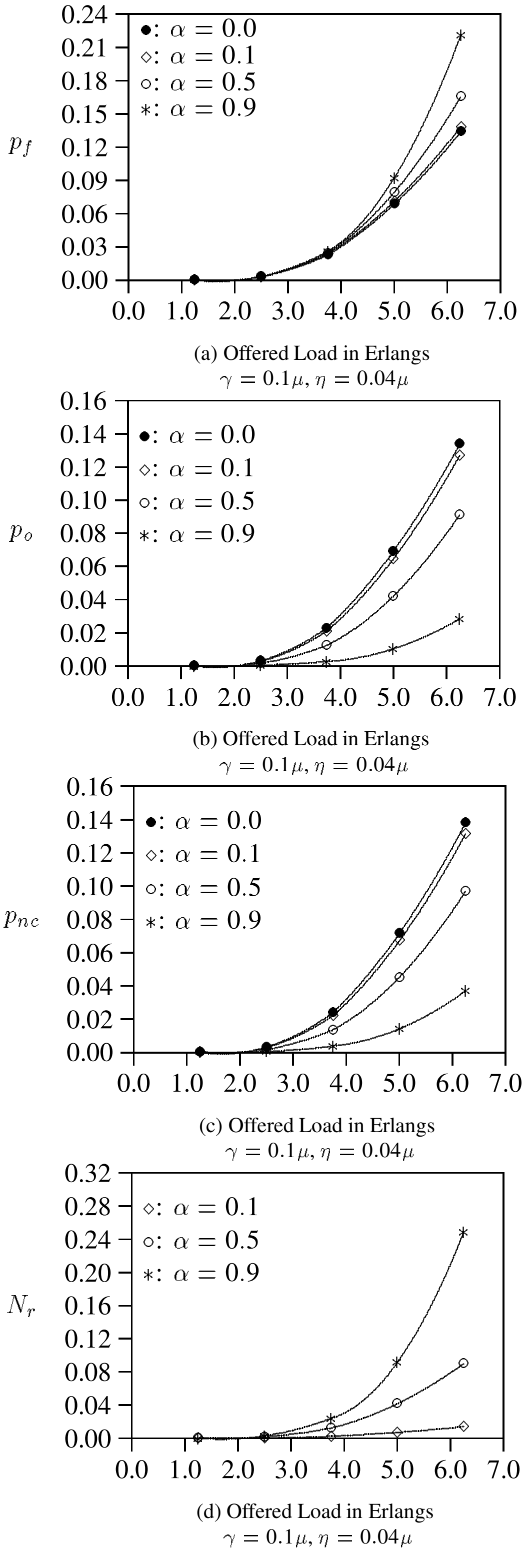

Figure 1: Effect of α on (a) p_f, (b) p_o, (c) p_{nc} and (d) N_r

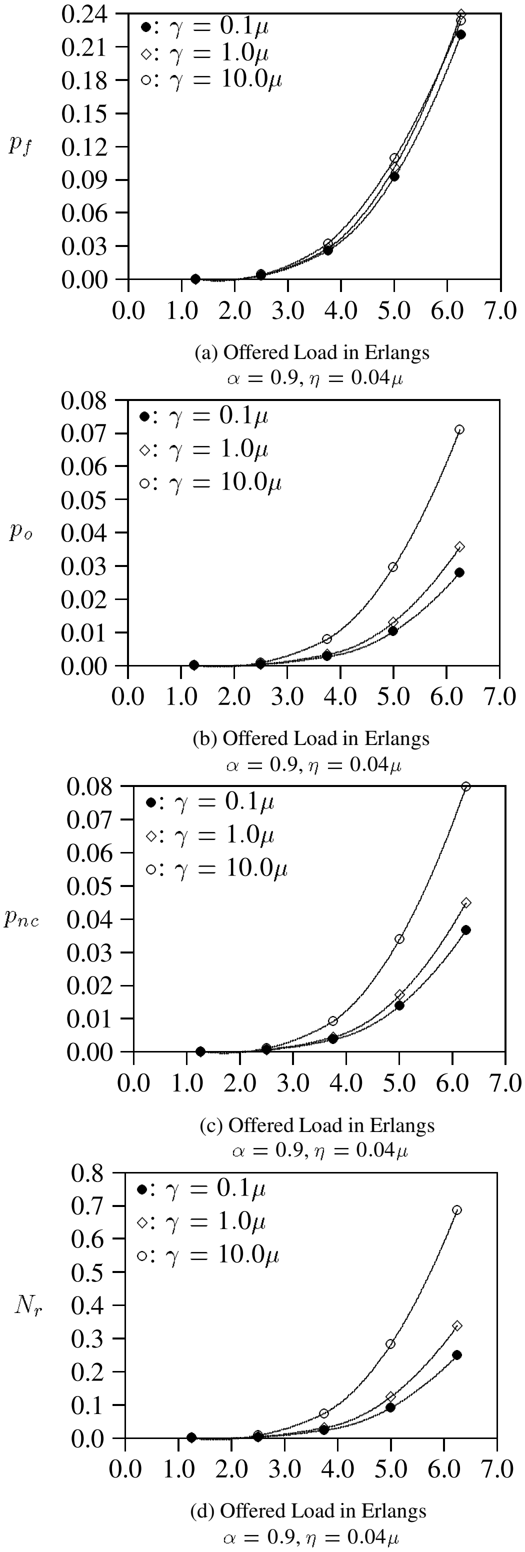

Figure 2: Effect of γ on (a) p_f, (b) p_o, (c) p_{nc} and (d) N_r

As explained above, when γ increases, the channel availability observed by a re-dial call decreases, resulting in a call having to make more re-dial attempts before a channel becomes available.

In Figure 3, the effect of η is shown. It is observed in Figure 3(a) that as η increases, p_f decreases. This effect is because as η increases, the length of time a call occupies a channel before moving decreases, which increases channel availability for hand-off call attempts, causing p_f to decrease.

Figure 3(b) indicates the effect of η on p_o. Again, because as η increases, the length of time a call occupies a channel before moving decreases, the channel availability for new call attempts is increased, resulting in lower p_o statistics.

Now, since p_o and p_f increase as η decreases, one would expect that p_{nc} should increase as well. However, this is not the case, as shown in Figure 3(c). When η decreases, so does p_{nc}. The explanation for this phenomenon lies in the relation between changes and η and changes in p_f, shown by equation(1). As η increases, the number of hand-offs required before a call is complete increases. Thus, it is likely that a call is eventually forced to terminate even if p_o and p_f decrease.

For example, in figure 3(a), the difference in p_f from $\eta = 1.0\mu$ and $\eta = 0.20\mu$ is only about 20%. To obtain this 20% change in p_f, η must be changed 500%. Accordingly, to obtain relatively small increases in p_f requires huge decreases in η, which results in smaller $\eta \times p_f$ values, causing p_{nc} to decrease as η decreases.

Like, p_f and p_o, N_r also increases as η decreases (Figure 3(d)). This effect is due to a decrease in channel availability observed by new call attempts because the length of time a call occupies a channel before moving decreases as η increases. Accordingly, a new call attempt must make more re-dials before a channel becomes available, which results in higher N_r statistics.

V Conclusions

In this paper, we present a model for studying the effect of re-dialing on a personal communications services network. Based on this model, several analytical results are derived for different PCS network performance statistics. These results are then used to validate our distributed simulation. Using the distributed simulation we study the effect of (i) α, the probability of re-dial, (ii) γ, the re-dial rate, and (iii) η, the portable movement rate and examined each parameters effect on (a) the probability of forced termination, p_f, (b) the probability of new call block, p_o, (c) the call incompletion probability, p_{nc} (d) expected number of re-dials before a new call attempt is accepted or rejected, N_r. The primary results from this study are:

- Re-dialing, when done at a low rate ($\gamma < 1.0\mu$) and high probability ($\alpha > 0.5$) helps to reduce the call incompletion probability, p_{nc}.

- Re-dialing, when done with a high probability ($\alpha > 0.5$) can cause an increase in the probability of forced terminations, p_f.

- Our study indicates that re-dialing may improve the call incompletion probability in the workload range of interest. However, from the caller's view point, frequent re-dialing implies a poor quality of service. Thus, the

PCS designer should carefully select the radio port capacity that provides a POTS level of service quality to PCS subscribers, as well as limit the frequency and number of re-dial attempts.

- We expect that re-dialing will have a negative impact on PCS networks with a heavy offered load or high mobility of subscribers. However, in the parameter ranges of interest, we do not see this negative impact. Accordingly, this will simplify the design tradeoffs.

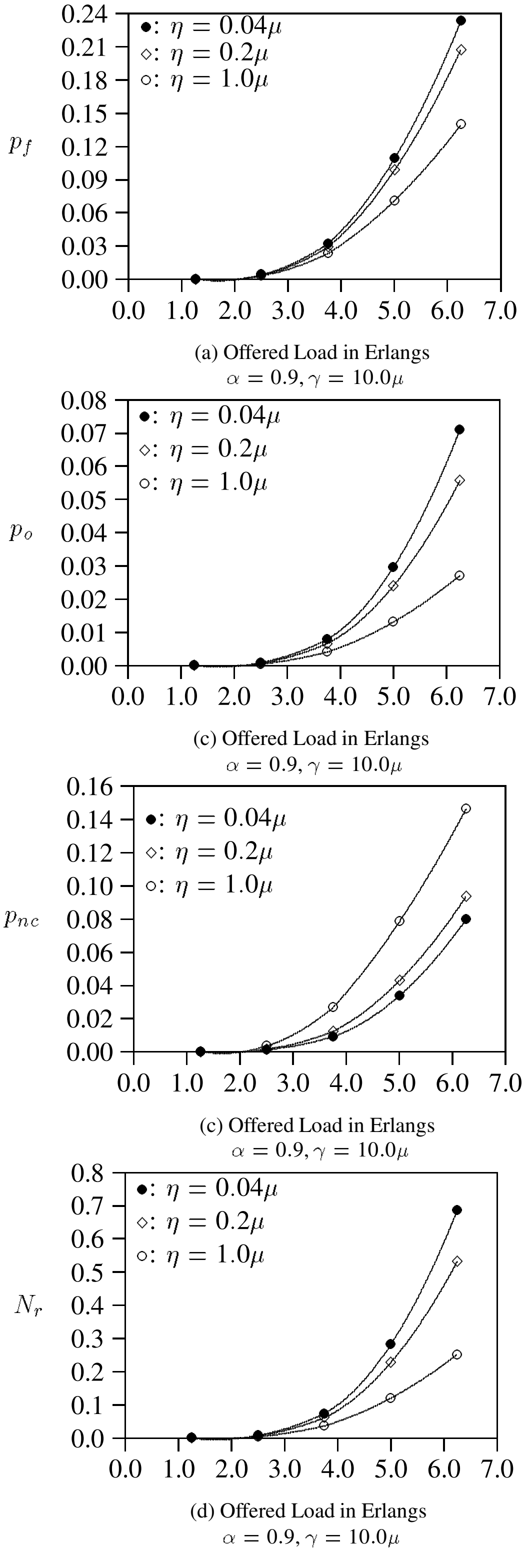

Figure 3: Effect of η on (a) p_f, (b) p_o, (c) p_{nc} and (d) N_r

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